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DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

THE YELLOW RIVER PROBLEM

By O. J. Todd,[1] M. Am. Soc. C. E., and S. Eliassen,[2] Assoc. M. Am. Soc. C. E.

Synopsis

The river problems of North China, aggravated by serious floods and heavy silt loads, have perplexed the Chinese for more than 4 000 yr. Especially has the Yellow River taxed the ingenuity of their hydraulic engineers, as shown by the numerous changes of course this river has taken across the Great Plain of North China through recorded history, changes that have meant appalling disasters to the population of the plain. What Western engineers have learned in recent years about its many-sided problems is outlined in this paper. Such evidence as is available is offered to indicate that certain phases of the Yellow River problem are much clearer to-day than they were in 1928 when the first attempts were made by Western engineers to probe into the hydraulics of this exceptional river and suggest methods for its control and regulation. Proposals for a general regulation program are also outlined, based on the most recent investigations. These proposals give promise of more adequate flood insurance for the population of the Great Plain than has existed previously. Since 1919, both writers have been connected with organizations, such as the Yellow River Commission, the Chihli or North China River Commission, and the China International Famine Relief Commission, all of which have been concerned with investigations of this river in a modern way. Data presented in this paper have been gathered by these Commissions, or under their direction unless otherwise mentioned.

Introduction

The Yellow River is unique among the larger rivers of the world in that, during freshets, it can carry a silt load that may reach 40% by weight where

Note.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **February 15, 1939**.

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it enters the alluvial plain of North China. The effect of this high silt load is to raise the river bed gradually, a phenomenon which can be divided into two distinct actions: (1) That in the upper reaches of the diked course, caused by the silt load itself; and (2) that in its lower reaches, due mainly to a flattening of the slope produced by the extension of the delta into the shallow Gulf of Chihli. The two actions, as far as they affect the rise of the river bed, overlap each other, and there is no distinct border line between them. Changes of the river's course through the delta may have far-reaching influences up stream in some years, much beyond the back-water influence of the tides; but the effect of such delta changes may be completely obliterated a year or two later, by the arrival of highly silt-laden freshets, or by dike breaches. The seaward growth of the delta is thus only partly responsible for the steadily increasing super-elevation of the river's course above the plain.

Locally, the gradual rise of the river bed (which averages about 3 ft per century) can often be as much, or more, in a few years time; but such deposition may again be partly scoured out during following years. This creates an exasperating river-control problem in that the dikes must be raised continually, sometimes as much as 5 ft or 6 ft, over lengths as great as 20 to 30 miles, and it may happen that dike sections recently raised must again be built up after a couple of years due to local deposition. It is an ever-continuing struggle to keep the height of the dikes ahead of the river. Often the river winds, overtops and breaks the dikes and, for a time, wanders unfettered across the densely populated plain causing immense damage and loss of life. Under such circumstances, it is impossible for the vast region on both sides of the Yellow River to develop.

The Yellow River flows on a ridge and is elevated above the plain through which it runs. Once it has broken its dikes, therefore, it will not, as a rule, go back to its low-water channel of its own accord when the flood is over. It remains outside and must be brought back to its course forcibly. This is a classical procedure in China and the person supervising it derives much credit if he is successful. If he fails, this is his misfortune. It may even cost him his life; but no glory accrues to the person who watches the dikes skillfully and keeps them intact. He is merely a "work horse."

That the Chinese have tried various methods of controlling this troublesome river, during the course of time, is evident, and the books and pamphlets dealing with its history and control are myriad; but, unfortunately, they have mostly been written by scholars who knew more about poetry than rivers. It is said of the great Emperor Yü (2200 B.C.) that he solved the problem of getting the river to flow in a bed below the surface of the plain. This skill seems to have died with him, however, and ever since attempts to do so have failed. Since Yü's time discussions regarding control methods have generally fallen into two groups—the diked single-channel method, and a system of multiple channels combined with irrigation and broad flooding when the freshet season is on. In practice, it has usually been a combination of the two methods. During the last ten centuries the single-diked channel method has been adhered to as far as the river has allowed.

The modern conception of channel contraction to induce a more concentrated flow with greater velocities to scour the channel is not new to the Chinese. History tells of much heated discussion among river engineers regarding the good and bad points of a single channel and a system of several channels. In the Ming Dynasty (A.D. 1368–1644), Pan Chi Hsun was the outstanding proponent of the single-channel theory. Already in his time the river was mainly held to a single, diked channel; but he had more advanced ideas and wanted a channel as narrow as possible in order to carry the silt and the floods to sea. His more conservative opponents advocated a wide distance between the dikes for storage of silt and for dissipating the floods. They said:

> "If the dikes are far apart there will be fewer places where we must defend the dikes by costly works and the dangers will be few. If the dikes are too close together the foreshore is easily washed away, the dangerous places will be many and the cost of protecting the dikes will be very great."

This argument holds to-day and has been defended by Western engineers, as witnessed by the proposals of the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E.,[3] by German engineers who have later visited the Yellow River, such as Professor L. Franzius, and by the laboratory experiments made in Germany for the Chinese Government in 1932 and 1934, experiments which do not entirely uphold the narrow-dike, distance theory.

One of the real obstacles to progress, of course, lies in the lack of hydraulic knowledge of the river itself. Until a relatively few years ago very little hydraulic data existed. This is now being overcome and gradually "light is being shed" on many of the perplexities that have shrouded the river in mystery and have made it subject to more superstition, perhaps, than that with which rivers in China generally are endowed.

There is probably no river in the world which is of so little use to mankind as the Yellow River considering the populous districts through which it flows. Thus far, people have been only partly successful in protecting themselves from its ravages. Even as a communication artery it is unimportant. The river is an enemy instead of a helpful agent. The task of making it obey the commands of Man is a most fascinating challenge to the Engineering Profession.

Scope of Investigations

Former Studies.—For more than half a century foreign engineers and others of scientific leanings have interested themselves in the problems of the Yellow River. The great breach that occurred in Eastern Honan in 1851 (see Fig. 1), causing this river to leave its southeasterly course to the Yellow Sea, and changing it permanently to a northeasterly direction emptying into the Gulf of Chihli, 400 miles to the north of its old mouth, was of pronounced significance. It was the year 1868, however, before a foreign investigator visited the site and prepared notes for publication telling of the effects of this channel change and discussing the various problems involved.

[3] "Flood Problems in China," by the late John R. Freeman, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1405.

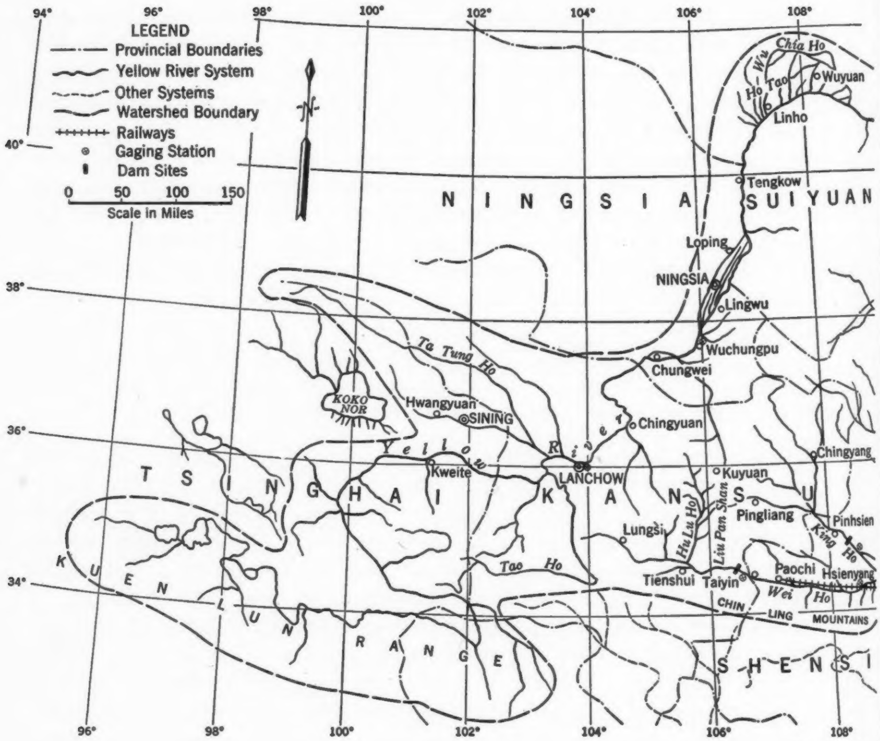
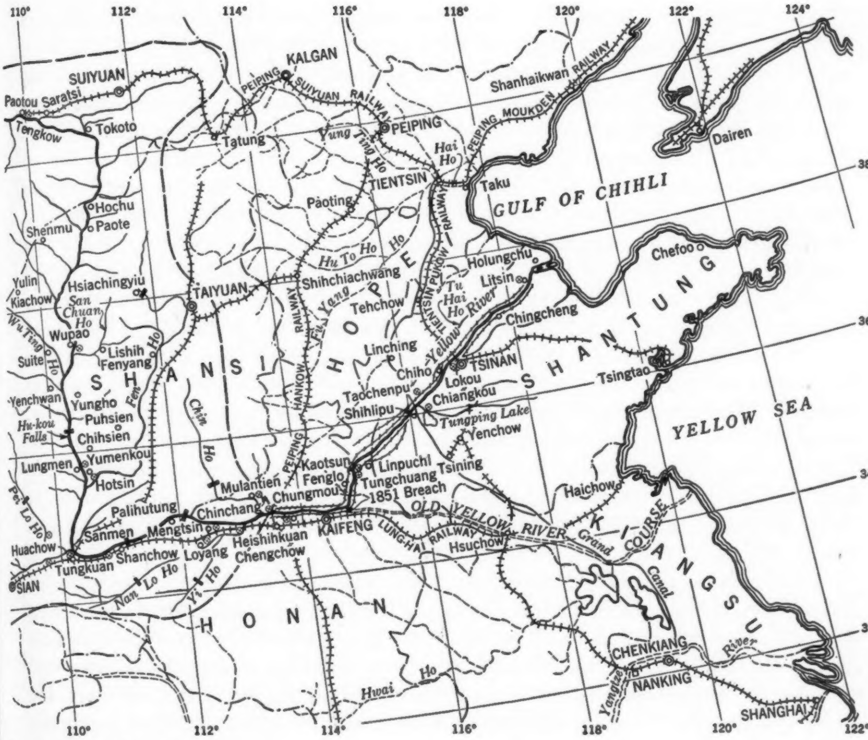


FIG. 1.—THE

The studies of the break of 1851 and that of 1887 by Ney Elias and, later, by G. J. Morrison (British), Gen. J. H. Wilson (American), Messrs. Shermbeck and Visser (Dutch), and others at various times over a period of 25 yr from 1868 to 1893, need no special remarks. The findings of these engineers and scientists, as well as those of Capt. W. F. Tyler in 1902, have been commented upon by the late Mr. Freeman.[3]

The mapping work made in the upper reaches of the river by European travelers and geologists such as Richthofen, Tafel, and Filchner (Germans), Obrutschew (Russian), D'Ollene (French), and others, are of much geographical importance, being the main sources for defining the western drainage boundaries of the river. The geographical findings of Pumpelly, Richthofen, Baily Willis, and lately of the Chinese and foreign geologists of the China Geological Society, are of the greatest value for understanding and furthering the engineering problems of the Yellow River.

Among the engineers of modern times, Mr. Freeman must be cited as the first who has gone into the flood problems of the Yellow River with such zeal and intelligence as to make his paper on "Flood Problems in China"[3] a contribution of outstanding value. He had devoted much time to a careful study of all the more important records extant. In connection with the Grand



Yellow River System

Canal improvement, he had plans which he came to China to supervise and complete, an unusual opportunity to have certain important data gathered, tabulated, and mapped. It may fairly be said, therefore, that the systematic investigation of the flood problem of the Yellow River, in a modern sense, began with Mr. Freeman.

Studies in Recent Years; General Surveys.—Prior to the autumn of 1933 the control of the Yellow River, where it ran through the plain, had been in the hands of the Provincial Governments of Shantung, Chihli (later Hopei), and Honan (see Fig. 1). These Governments had their own survey systems, differing in co-ordinates, scales, and elevation datums. Independent of each other they managed as well as possible, and the political and financial questions to be solved generally outweighed the technical ones. However, one Chinese Government organization in Tientsin (the Chihli River Commission, which was investigating the river problem of the Province of Chihli on account of the silt and flood situation affecting the City of Tientsin and its navigable river, the Hai Ho) took an active interest also in the Yellow River since this river had a direct bearing on the port problem of Tientsin. Centuries ago, the Hai Ho (river) from Tientsin to the sea was the outlet of the Yellow River into the Gulf of Chihli, and it is not unlikely that it again may take this course.

The Chihli River Commission, therefore, began its own independent studies in 1919, had its engineers run level lines down to the Yellow River at various points, and established two hydrometric stations on the river, one at Lokou, 150 miles from the sea, and one at Shanchow, in Western Honan, 600 miles from the sea. Although involving the observation of the river only at two places the data collected have been very valuable in that, at least as far as water-level observations are concerned, they have been uninterrupted since 1922 in spite of all civil wars, and they have been continued to date (1938) by the Yellow River Commission, a Chinese National Government organization which was established in the autumn of 1933. (In all recent studies of the Yellow River the so-called Taku Datum (T.D.), named after the Village of Taku at the mouth of the Hai Ho which passes through Tientsin, has been used as elevation datum. It is a zero reference plane corresponding to the low water of spring tides at the mouth of the Hai Ho. The Grand Canal Improvement Board, in its studies of the Grand Canal and the Yellow River, used the so-called Tsingtao Datum which corresponds approximately to zero elevation of extreme low tides of the ocean at Tsingtao Harbor. The relation between Taku Datum and Tsingtao Datum near Lokou is approximately: Taku Datum minus 5.25 ft equals Tsingtao Datum. Taku Datum is 4.50 ft below mean sea level.)

The hydrometric investigations by the Grand Canal Improvement Board during the spring, summer, and autumn of 1919 covered water level, discharge and silt observations, and studies of scour and refill of the river bed at two sections near the Grand Canal crossing. Furthermore, its engineers surveyed the dike system of the Yellow River between the Peiping-Hankow and Tientsin-Pukow Railways, all being engineering data (including cross-sections) pertinent to the Grand Canal improvement problem.

Between 1929 and 1932, the China International Famine Relief Commission systematically collected hydrometric data at a point on the Yellow River 12 miles down stream from Paotou in Suiyuan Province in connection with the Saratsi Irrigation Project. Other irrigation projects constructed between 1930 and 1935, drawing water supply from tributaries of the Yellow River (such as from the King River in the Province of Shensi and the Fen River in the Province of Shansi), naturally necessitated the study of these rivers and the knowledge thus gained contributed to the general understanding of the Yellow River problem.

A more ambitious program of investigations has been conducted by the Yellow River Commission since October, 1933. This Commission has conducted hydrometric studies at fifteen stations along the main river and at sixteen stations along the more important tributaries. These stations are shown in Fig. 1. In addition to the hydrometric work a detailed topographic and hydrographic survey has been made of the entire course of the Yellow River through the plain, including the sea coast outside the delta. This survey, mapped to a scale of 1 : 10 000, with contour intervals of 1.64 ft (0.5 m), covers the area between the dikes and approximately a 5-mile strip outside. The total area surveyed is about 9 500 sq miles. It has been linked to the topographic survey of the former Chihli River Commission, later the

Hua Pei River Commission. This was easy to do because the survey of the Yellow River Commission has been patterned closely after that of the Chihli River Commission.

In addition to the more precise mapping work in the plain reconnaissance, surveys have been conducted along the upper reaches of the river from Lanchow in Kansu down stream to Hochu in Northern Shansi, a distance of more than 1 000 miles.

Hydraulic Laboratory Experiments.—In order to reach some conclusions regarding the much-discussed question of increased erosive force of the river if the distance between the dikes were decreased, the Chinese Government had certain hydraulic laboratory experiments conducted in Germany in 1932 and 1934. The first set of experiments in 1932 were made with the channel straight and the dikes set back so that the width ratio between foreshore : channel : foreshore was approximately 3 : 2 : 3; whereas for the "narrow-dike-distance" experiments, the dikes were set close to the edges of the channel. In the experiments of 1934 the channel was made moderately winding. In one of the experiments the dikes were made straight and were located so that they nearly touched the outer concaves of the winding channel, and in the second experiment they followed the winding channel closely. In the first of the two winding-channel experiments, low, straight "wing-dikes" had been built out from the main dike, and followed one bank along the tangents which connected the channel bends, in order to guide the flow as much as possible along the curved channel, thus preventing it from flowing across the foreshores except at high stages. The idea was to concentrate most of the flow along the curved channel, with relatively sluggish flow over the foreshores, so that deposits would tend to form there, whereas the channel would be scoured.

Although no very decisive conclusions could be drawn from the experiments the results gave some startling indications. With the dikes set far apart a somewhat better scour was produced in the channel (with a considerable amount of silt deposition on the foreshores) than when the dikes were close to the channel edges. In the experiments with curved channels the wing-dikes seemed to have had the desired effect of silting the foreshores without raising the bed of the channel; in fact, a slight bed scour was discernible. Noticeable silting of the channel occurred, however, in the experiments with the dikes following the edges of the channel closely. It should be remarked that the kind and quantity of silt used, the length of the flood and low-water season, and the intensity and duration of the flood flow, were assumed quantities and bore little relation to those of the Yellow River. Therefore, the experiments indicate only, in a general way, what the results are likely to be if dikes are built close to, or far from, the banks of a silty river.

When the results became known they had the effect of postponing any moving of the present dikes closer to the banks of the river. A highly silty river does not seem to act quite according to customary hydraulic assumptions or calculations, and in this respect the experiments were illuminating. Since they were made the hydraulics of the Yellow River had become better known, and further laboratory experiments may give more specific results regarding the reaction of the river to contemplated control measures.

Soil Erosion Experiments.—Base surveys have been made of gullies and sloping lands where the erosion is severe, with a view to conducting experiments with soil retention measures, both structural and agricultural, on sloping lands as well as in the gullies. Since the silt is the "stumbling block" to nearly all conservancy measures on the Yellow River, it is evident that, in the future, soil-erosion control will assume greater and greater importance.

Former Attempts to Cope with the Flood Problem

Such records as are trustworthy show that for many generations (at least during the past 1 000 yr) the Chinese have depended chiefly on their dike system as a means of controlling floods on the densely populated Great Plain. Inspection of old banks of former river courses, such as those near Techow in Northern Shantung, where the river ran during the Tenth and Eleventh Centuries A.D., indicates clearly that earth dikes were the principal means of keeping this troublesome stream within bounds. Likewise, its control was within one channel for the 500 yr preceding the break of 1851, as effected by a single line of strong dikes. Practically no evidences remain of stone work being used to protect this dike system, of which plain remnants exist below the point of the break 30 miles east of Kaifeng. Records reveal that when dikes did give way in places the breaches were repaired by a prodigious use of kaoliang stalks, hemp rope, willow stakes, bags of clay, etc. This method of closing a breach has gradually been improved and is used with certain modifications to this day.

Ancient history, perhaps mythical, reveals that the great Emperor Yü divided the flow across the plain into nine channels and by this method succeeded, as mentioned in the "Introduction," in getting the bed of the channels to lie below the surface of the plain. There is evidence to support the belief that the serious erosion period which Northwest China now experiences began about the time of the great Yü and, if this is correct, it is most likely that he would not have had the silt masses to contend with which now are such obstacles to an effective control of the Yellow River. Undoubtedly, it was the increasing silt masses which finally destroyed Yü's work. Since then pandemonium has reigned. Many control methods have been tried to prevent the river from raising its bed, but to no avail. In 1073 A.D., during the Sung Dynasty, a proposal was made by an official and scholar, Wang An-Shih, in which he urged that 1 000 iron scarifiers be used to stir up the mud in the river bed at low-water stages so that it might be carried down stream and the channel thus lowered. His idea was not adopted because it was thought to be too costly and not very promising in result; but it is perhaps the first recorded proposal for dredging in the history of the Yellow River. Pan Chi Hsün and his dike contraction idea, which created such a controversy, has already been mentioned in the "Introduction." On a stone tablet which to-day can be seen on top of Yün Lung Shan (outside the City of Hsüchow, at the crossing of the Tientsin-Pukow and the Lunghai Railways, and on the bank of the old Yellow River course previous to 1851) Pan Chi Hsün has left an inscription which bemoans the fact that his ideas were not understood.

By the middle of the Nineteenth Century the bed had become so high that when the breach of 1851 occurred, it was found impossible to close it, especially as the Taiping Rebellion was raging at the time. The breach is known to the Chinese as the Tung Hwa Hsiang Breach. Due to the great difference in elevation between the river bed and the land outside the dikes the flow rushing out through the breach caused a back-cutting toward the up stream which lowered the river bed for a considerable distance. For more than 25 miles up stream from the breach the foreshore lands now (1938) have elevations which are still higher than the highest flood levels since 1851; and from the Peiping-Hankow Railway Bridge to the Shantung border the river is still actively adjusting its grade as a result of this breach which occurred 87 yr ago.

After ten years of indecision the new course toward the northeast was finally accepted as the new permanent channel of the river; but nearly thirty years elapsed before an adequate dike system had been built which could be said to control the river during years of low flood flow; and even to-day there is great difficulty in keeping a medium-sized flood from overtopping or breaking through the dikes in the section between the 1851 break and the Shantung border.

The most notorious breach that has occurred since the Tung Hwa Hsiang catastrophe is perhaps the one in 1887. It occurred more up stream, between Kaifeng and the Peiping-Hankow Railway Bridge, and, as a result, for two years, the river flowed southeastward to the Huai River and the Hungtze Lake before the breach was finally closed during mid-winter when the flow

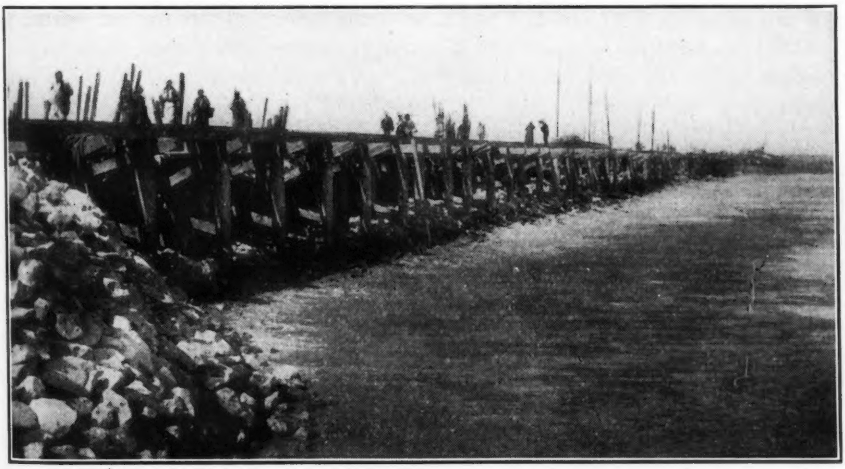


Fig. 2.—Completing Rock-Fill Dam from Trestle Across the Yellow River to Divert Flow Back into Its Old Bed, April, 1923

was exceptionally low. Another serious break occurred in 1921 in Eastern Shantung near the coast when the entire flow of the river left the old bed. The successful closure of this breach, by means of a rock-fill dam, was reported in 1933[4] (see Fig. 2).

[4] "Wrestling with Some of China's Rivers," by O. J. Todd, *Civil Engineering*, June, 1933, p. 304.

The next major break in the main dikes occurred in August, 1925, in Western Shantung One of the writers made surveys and estimates during the following months and drew up a plan for closure work. The Shantung Provincial River Bureau effected a closure within six months. Fortunately,

FIG. 3.—ICE JAM IN YELLOW RIVER JUST ABOVE LITSIN, SHANTUNG, FEBRUARY, 1928

on this occasion, only one-half the flow of the river had left its bed, running into a low area that emptied into Tungping Lake and from there back to the Yellow River, 75 miles east of the break. It was in this region that the break occurred in 1919 as described by Mr. Freeman.[5]

In late February, 1928, a heavy ice jam caused ponding and overtopping of the south main dike in two places east of Litsin, Shantung, near the coast (see Fig. 3). This breach was closed within a few months using old Chinese methods; but considerable aid was given by the river itself in that it partly silted up the breach openings. It was in this vicinity, but 10 miles up stream on the same bank, that the dikes failed in August, 1937, during heavy, prolonged summer floods. The region then inundated was estimated at 1 000 sq miles, approximately equal to that of 1928. No attempt has been made to close this breach. The country affected is not densely populated and the land has much alkali. It lies in the lower delta region close to the sea, and sluicing silt on it will convert it into good, arable land, after a few years.

Dike breaks occurred in 1933, 1934, 1935, and 1937. In flood magnitude the break of 1933 was the most serious in that the highest flood, perhaps since 1842, then occurred. Between the place of the Tung Hwa Hsiang breach and the Shantung border the dikes were breached in thirty-five places. Several long sections of the north dike were buried in silt so that nothing could be seen except the two lines of trees that grew along the top of the dike. Due to the extremely heavy silt flow of the river the breaches gradually closed

[5] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), Fig. 5, p. 1414.

themselves except the one farthest down stream on the north bank. This breach, called the Fenglo breach, was closed the following spring by gradually contracting the opening, working from both sides. Originally, it had been planned to build a wooden pile trestle across the opening and close it by stone-dumping from this trestle, a method similar to the one so successfully used in 1923. The piles were driven before the break up of the ice, however, and subsequently the ice flow sheared off all the piles. The contraction method was finally resorted to and "one-man-sized" rock enclosed in woven willow casings, usually more than a cubic yard in volume, were used to close the final 50-yd opening. In its last phases, the work was greatly facilitated by the silting action of the river itself during a small spring freshet.

The 1933 flood had destroyed so much of the dike system that when the flood season arrived n 1934 only a 10-mile section of the dike, abutting the northern end of the Tung Hwa Hsiang breach, remained to be rebuilt. Since there was an outer dike (the Golden Dike) which, in case of overflow, would deflect the escaped water back into the river near the Grand Canal crossing, the situation was not considered serious. However, 1934 also proved to be a year of high floods and, in August, a side channel toward the north gradually developed across the former dike line which had been buried in silt; and by the time the flood season was over in October the side channel was taking nearly half the flow and threatened to become the main river.

FIG. 4.—A VILLAGE IN SOUTH HOPEI BURIED IN SILT BY A YELLOW RIVER DIKE BREAK, MARCH 1, 1935

The Golden Dike was seriously endangered and if it should break the flow would most likely go to Tientsin. A major closing operation became necessary. An attempt was first made to contract the opening sufficiently by layers of kaoliang stalks and earth so that a plug of the same material could be lowered into the opening. This could not be wider than 80 ft if one were to be sure of success. Unfortunately, the river bed proved to be very sandy to great depths and became deeply scoured when the opening had been contracted to

Fig. 5.—Building Kaoliang Core of Closure Dike at the Great Breach near Tungchuang, Shantung, Caused by the 1935 Floods

Fig. 6.—Tying Up Willow Cased Stone "Sausage" to Roll into Last Gap in Closure Dike at Tungchuang Breach, March, 1936

FIG. 7.—PLACING LAST STONE "SAUSAGES" TO CLOSE BREACH OF 1935, AT TUNGCHUANG, SHANTUNG, ALONG SOUTH BANK OF YELLOW RIVER

FIG. 8.—WILLOW STONE "SAUSAGES" AT TUNGCHUANG CLOSURE HELD IN POSITION AGAINST HEAD OF 5 FEET OF WATER BY THREE OR FOUR TWISTED WIRE CABLES AFTER HEMP ROPES FAILED

about 120 ft. The ends of the closing work became undermined, broke off, and floated away like great haystacks, and no further progress could be made by this method. Soundings showed depths of nearly 70 ft. Finally, the river bottom was sealed by the use of long fascines made of woven willow matting wrapped around an inner core of rubble and tied together with hemp rope. As in the case of the closure work at the Fenglo breach, the river threw a silt bar in front of the opening during a silty spring freshet and the final closure work could practically be done in quiet water.

The break at Tungchuang, in Western Shantung, the following year (1935) proved by far the most disastrous since the break of 1887, because it inundated approximately 6 000 sq miles of farming country in Western Shantung and Northern Kiangsu, the flow going to sea at a point near the former mouth of the river during the five centuries prior to the change of course in 1851. Fig. 4 is a view, taken March 1, 1935, of a village in South Hopei. Before the end of 1935 the entire flow was pouring to the southeast through the breach. The Shantung Provincial Yellow River Bureau began the work of closure; but after two months of preliminary arrangements the National Government set up an organization that assumed charge of the task, and completed the closure successfully by the end of March, 1936. In connection with this work the writers were engaged as Assistant Chief Engineer and Consultant, respectively. Benefiting by the experience gained during the Fenglo and Kuant'ai breach closure works (the Chinese refer to the 1934 breach or side-channel closure as the Kuant'ai work), stone was used in much larger quantities, and after the breach opening had been contracted to about 130 ft, with a maximum depth of about 40 ft, work was begun immediately to seal the river bed and prevent it from being scoured further (see Figs. 5, 6, 7 and 8). The fascine "sausages" were made about 3 ft in diameter and from 40 to 60 ft long, and they were considerably larger than those used at Kuant'ai. The discharge during the time of closure averaged 30 000 to 35 000 cu ft per sec, but in spite of this rather heavy low-water flow the work proceeded without any serious setback. In the last three days, during the final closure, the river had to be "headed up" more than 6 ft in order to force the flow back into the old channel. This was facilitated by a cut-off channel 3 miles long which short-circuited a 10-mile sharp bend immediately below the breach and acted as a "primer" for accelerating the scour action in the old channel.[6] Fig. 9 shows a stone rip-rap apron with a woven willow cover, tied back by twisted wire ropes to hold small stones in position until silt fills the voids. A kaoliang wall tops the dike with 2 ft of earth cover. Fig. 10 shows a bent protected by a loose brick toe held by wire mesh.

Physiography

The Yellow River has its origin in a series of lakes and swamps on a plateau in Eastern Tibet. Before it reaches China proper it has cut its way through the mighty mountain ranges of the Kuenlun in a series of magnificent gorges. Reaching the Province of Kansu it receives its first important tributary, the Tao Ho, from the southeast. Then passing Lanchow, the capital of Kansu,

[6] *Engineering News-Record*, May, 1936.

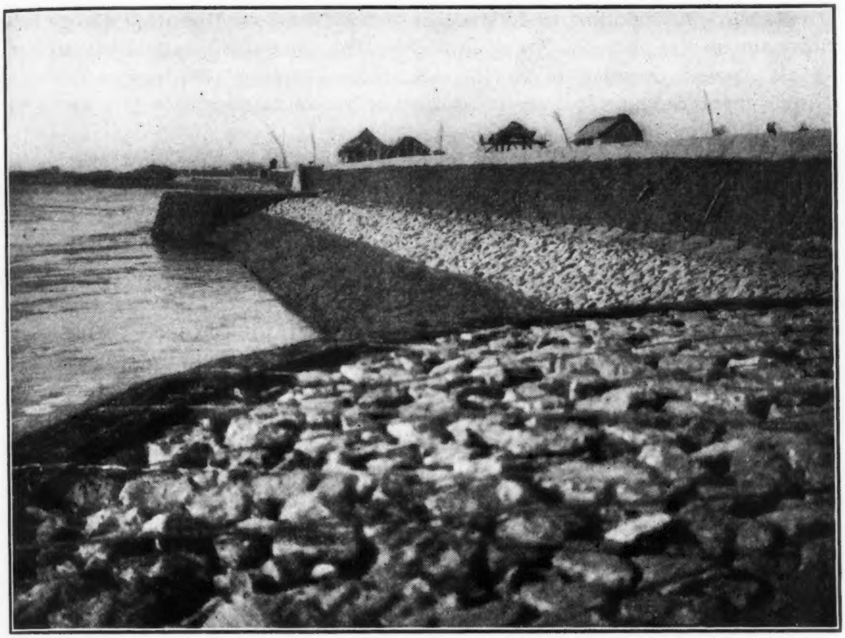


FIG. 9.—FINAL PROTECTION WORK TO CLOSURE DIKE AT TUNGCHUANG, STONE RIP-RAP APRON WITH WOVEN WILLOW COVER TIED BACK BY TWISTED WIRE ROPES TO HOLD SMALL STONES IN POSITION UNTIL SILT FILLS VOIDS. A KAOLIANG WALK TOPS DIKE WITH 2 FEET OF EARTH COVER.

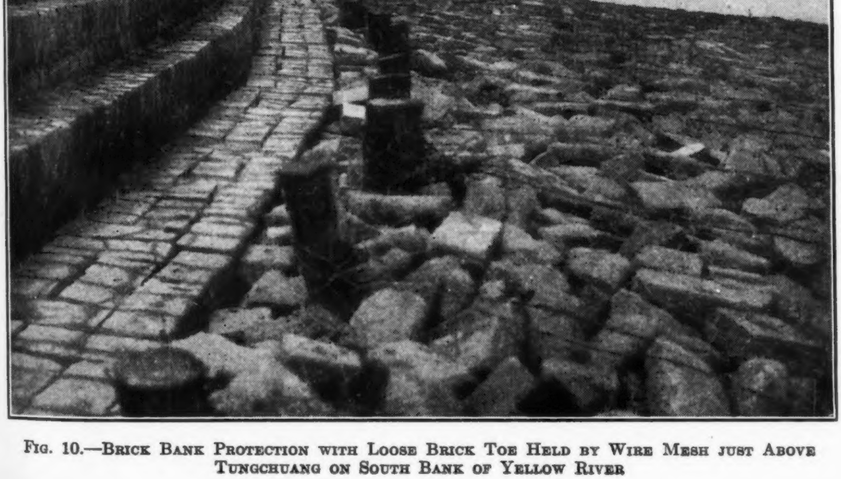


FIG. 10.—BRICK BANK PROTECTION WITH LOOSE BRICK TOE HELD BY WIRE MESH JUST ABOVE TUNGCHUANG ON SOUTH BANK OF YELLOW RIVER

it cuts through the most easterly spurs of the Nanshan Mountain Range and flows out on the plains of Ninghsia. Here, the course widens and islands and shoals appear everywhere in the channel. Forming the border between Ninghsia and Suiyuan it is again hemmed in by low mountains before emerging on the Suiyuan Plain.

It is typical of all the rivers in North China that they flow through a series of gorges and basins, the slope being flat through the basins and steep through the gorges. The basins are overflow and depositing areas for the loessic silt which the rivers carry in freshets and the gorges are cut through denuded mountain ranges where the rocky surfaces are exposed. The main basins of the Yellow River are in Ninghsia and Suiyuan, and between Lungmen and Tungkuan. However, there are many smaller, intermediate basins, such as at Chungwei, in Kansu. The alluvial plain of North China may be regarded as the last basin. The gorges first extend from the plateau near the source to Kansu, through which Province the river runs in a series of shorter gorges. Where it follows the Ninghsia-Suiyuan border it has a shallow, but remarkably steep, sloping gorge. From the Suiyuan-Shansi border to Lungmen the entire course is gorge like, and, finally, the last gorge extends from Shanchow to where the river enters the alluvial plain of North China. The slope through the gorges is often steeper than 1 : 1 000 with occasional rapids and waterfalls; through the basins (such as the Suiyuan Plain) the slope may be flatter, locally, than 1 : 10 000, but usually it ranges from 1 : 2 500 to 1 : 3 000.

Through Ninghsia and Western Suiyuan, for a distance of more than 400 miles, the river has practically no tributaries. This region is arid, with an average rainfall of scarcely more than 5 in. per yr; but, as it leaves Suiyuan and enters the north-south course between Shansi and Shensi the river receives numerous tributaries from both sides. These carry little flow during the dry season from November to June, but during the rainy season in summer they may be raging silt torrents. Then, at Tungkuan, where the course bends abruptly toward the east, its most important tributary—the Wei Ho—enters. The Wei Ho has many small tributaries draining the lofty Chinling Mountain Range which flanks it closely on the south. From the north, draining the loess plateau, it has only a few, but highly important tributaries. From Tungkuan to where the Yellow River enters the plain the tributaries are also numerous, but, as is the case between Tungkuan and Suiyuan, they carry almost no flow during the dry season whereas in the rainy season they are of much importance. Especially notorious flood carriers are the two streams, the Lo Ho from the south and the Chin Ho from the north, which enter the main river just up stream from the point where the dike system begins.

Besides the Chinling Mountain Range which forms the divide between the Yellow River and the Yangtzekiang drainage areas, the Liu Pan Shan Mountain Range in Southeast Kansu should be mentioned. This range has a north-south direction. It stands directly in the way of easterly rain-bearing winds and thus causes copious, orographic rainfall on the head-waters of the King Ho, the most important tributary to Wei Ho from the north. Table 1 gives the distances and elevations of a number of points along the river, which have been definitely connected by surveys and leveling.

The drainage area of the Yellow River above the Peiping-Hankow Railway Bridge is 294 000 sq miles (756 000 sq km). Some geographers like to include also the tributaries that belonged to it when it had a different course to the sea than it has now, for example, when it was flowing north to Tientsin or south to the Yellow Sea. In such case, its drainage area would be increased about 65%; but at least for the time being one must assume that the river will most likely be kept to its present course for another century. In that case its area can only be regarded as lying up stream from a point where it enters the fully diked course at the Peiping-Hankow Railway Bridge. To this should be added the Shantung area of about 4 500 sq miles. The exact value to be given to the last area is difficult to determine because during floods the flow becomes divided and much of it does not enter the Yellow River, but flows south into the Grand Canal and the West Shantung Lakes.

TABLE 1.—PROFILE ELEVATIONS OF LOW-WATER FLOW IN THE YELLOW RIVER

Name of place (see Fig. 1)	Elevation of low-water flow, in feet T. D.†	Distance from the Gulf of Chihli, in miles	Slope	Name of place (see Fig. 1)	Elevation of low-water flow, in feet T. D.†	Distance from the Gulf of Chihli, in miles	Slope
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Mouth of Yellow River	4	0		1851 Breach	228	365	
			1 : 6 000				1 : 5 300
Holungchu	28	27		Railway Bridge‡	305	441	
			1 : 10 000				1 : 3 800
Litsin	36	42		Mengtsin	390	503	
			1 : 12 600				1 : 1 100
Lokou	82	152		Shanchow	952	624	
			1 : 9 000				1 : 2 800
Shihlipu*	131	236		Tungkuan	1 053	677	
			1 : 7 800				1 : 3 000
Tungchuang	179	307		Lungmen	1 198§	760	
			1 : 6 000				1 : 1 600
1851 Breach	228	365		Paotou	2 920§	1 275	

* Grand Canal Crossing. † T. D. = Taku Datum. ‡ Peiping-Hankow Railway Bridge. § Approximately.

The chief characteristic of the Yellow River drainage area is its loess and tertiary clay deposits, which are especially extensive in Eastern Kansu, Northern Shensi, and Shansi. They are found in the river valleys, between the mountain ranges, and exist as a continuous plateau extending from Eastern Kansu across Shensi into the Province of Shansi. Frequently, the deposits have depths of more than 300 ft. It should be noted that the aeolian loess (in Chinese, *huang tu*, or "yellow earth") is a relatively recent deposit. Hence, it is generally the top-soil cover. It is underlaid by the similar, but older, tertiary clay, which, as a rule, is of a more reddish color. Over large areas the loess cover has been entirely stripped off and the red tertiary clay is fully exposed (see Figs. 11, 12, and 13). The tertiary clay is a soil of finer texture than the loess and is more resistant to erosion. Therefore, rivers passing mainly through tertiary clay areas, such as the Honan Lo Ho, as a rule, are less silt-laden after heavy rains than those which pass through areas where the loess cover is still deep.

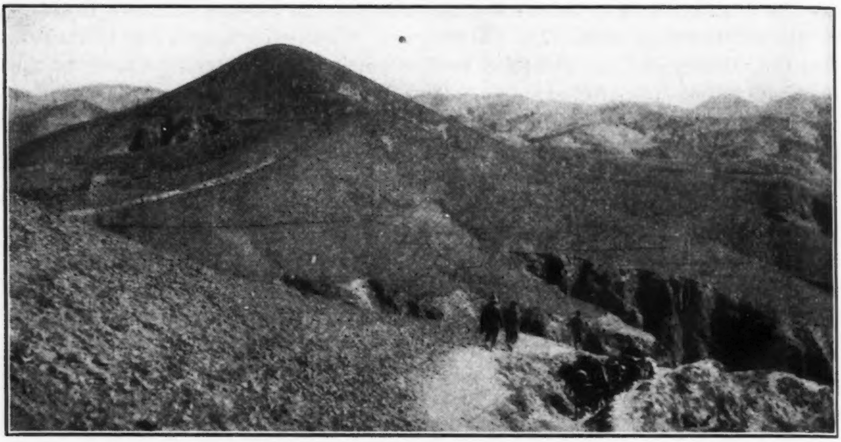


FIG. 11.—LOESS-COVERED HILLS OF CENTRAL SHENSI

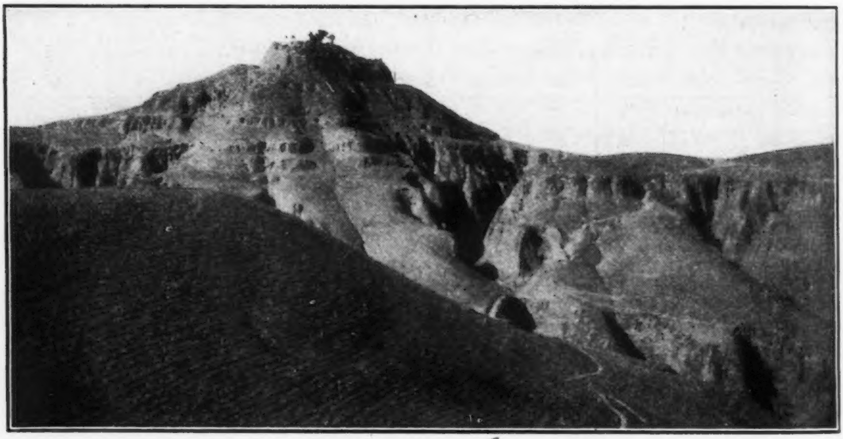


FIG. 12.—LOESS-COVERED HILLS OF CENTRAL SHENSI

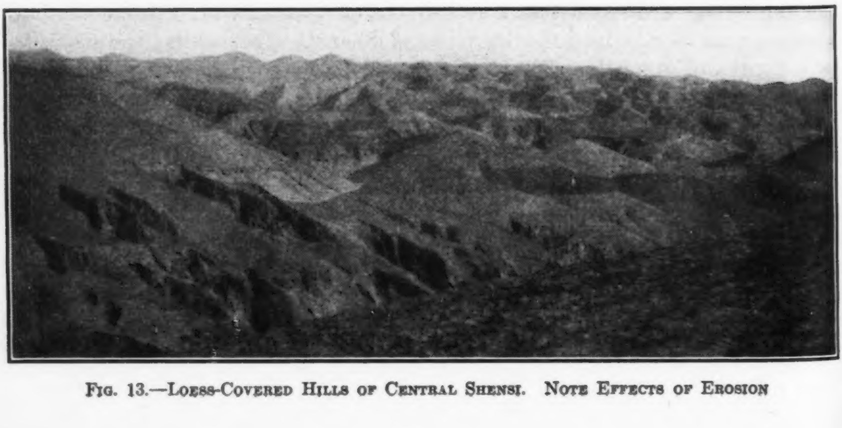


FIG. 13.—LOESS-COVERED HILLS OF CENTRAL SHENSI. NOTE EFFECTS OF EROSION

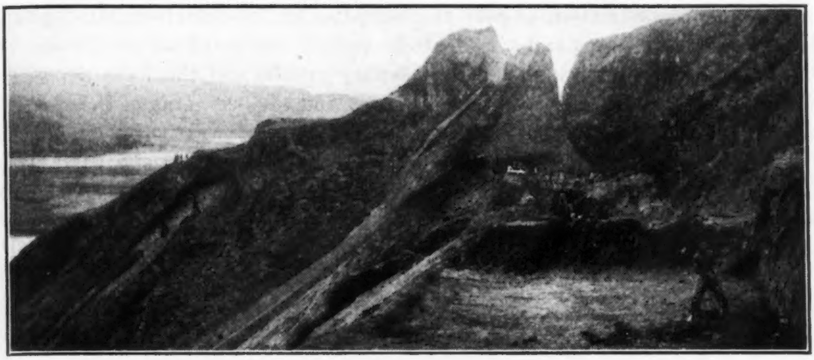


Fig. 14.—A Cut in Loess in West Shensi Along Motor Road from Sian to Lanchow, 1932

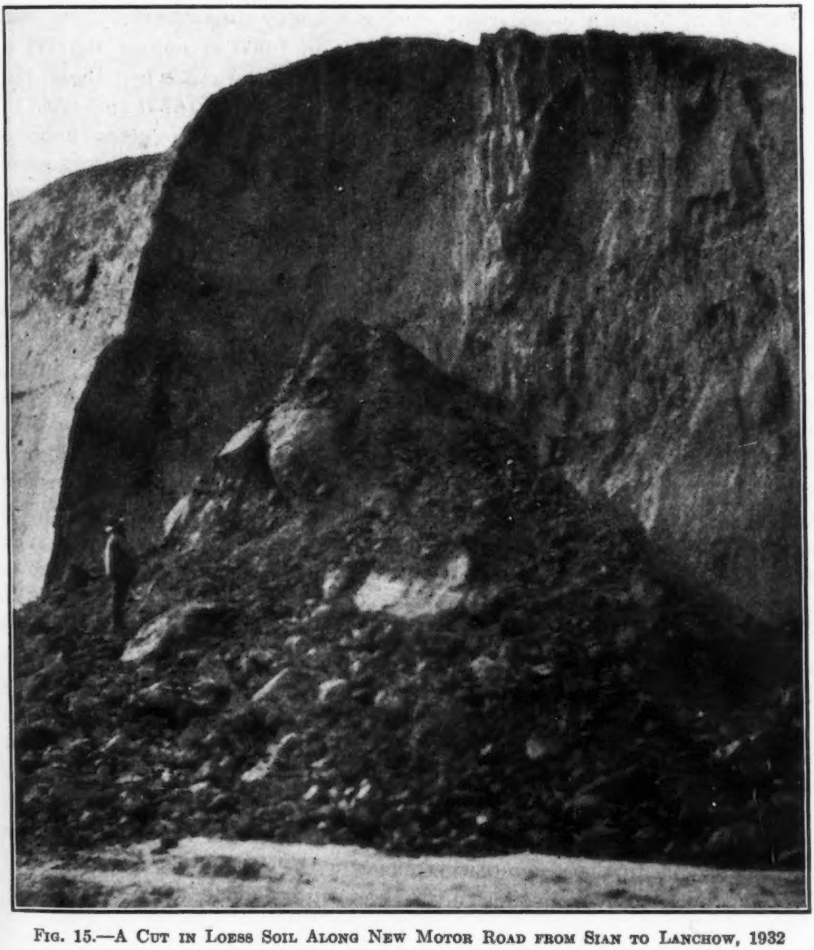


Fig. 15.—A Cut in Loess Soil Along New Motor Road from Sian to Lanchow, 1932

The origin of the loess, or even the tertiary clay, is a much disputed point. The tertiary clay seems to have filled the valleys as a result of the leveling of the mountain areas throughout the Tertiary epoch, and the loess deposited intermittently toward the end of the Tertiary and the beginning of the Quaternary ages, either blown in from the north or due to weathering of all the mountain ranges which traverse the Kansu, Shensi, and Shansi areas. This was a period of very wet, very dry, and windy, climate when erosion by stream flow and soil deposition by wind alternated. Hence, one may often find the strangest mixture of layers of gravel, loess, and tertiary clay in exposed, high, vertical river banks (see Figs. 14 and 15). Most likely, the present is a period of erosion, as witnessed by the heavy silt load which all the North China rivers carry in the rainy season; but the dust storms that occur in the spring are reminders of the earlier dry periods when the loess cover was laid down. Even now, in less than a day's time, a cover of nearly 0.01 in. of dust may settle on floors and tables in a closed room during a heavy dust storm. Two such storms per year would deposit a 20-in. layer in 1 000 yr and in 100 000 yr (which approximate the duration of the geological periods when these dust deposits were formed), the depth of the deposits would be 167 ft (modified by a suitable settlement factor), which is an average depth of the actual deposits on much of the Yellow River water-shed. As these dust storms always occur when there is a north gale, it seems that there is a gradual shifting of the soil toward the south by the wind; and, as all the rivers flow toward the east, there is also a gradual shifting of the soil toward the east by water action.

To-day the terrain in the loess and tertiary clay areas has become most severely cut by erosion. The farmers are fighting a losing battle against this destruction, and every year they see edges of their land worn away, or the top soil stripped off, by the heavy rain storms, which are so frequent during the rainy season. The entire area has been most extensively, but rather carelessly, terraced, to hold the soil in place, often with good result; but frequently, perhaps due to internecine wars, the terraces have been abandoned. For all practical purposes, the entire area may be said to be denuded of forests if there ever have been forests on it. Trees exist around villages and temples and on some of the more favorable mountain slopes facing northeast; but their effect is negligible in preventing the erosion which is now progressing at an alarming speed.

The Yellow River Basin and the surrounding area, especially its western part, are known as a great earthquake region. To the east its alluvial plain has been built up on a sinking geosynclinic trough, and earthquakes at times also shake this plain severely, as in the beginning of August, 1937. It seems reasonable to believe that the earthquakes are caused in part by the constant shifting of earth loads both by the Yellow River and by the wind from the mountain areas in the northwest to the plain in the east and southeast, amounting to an average probably exceeding 2 000 000 000 tons per yr. That isostatic adjustments due to the accumulating effect of the constant transfer of such huge loads must occur from time to time seems quite evident.

A. W. Grabau, of the China Geological Survey, holds the view[7] that the sinking of the geosynclinic trough, upon which the alluvial plain of North China has been formed, is caused by tectonic thrusts toward the east of the mountain masses of Central Asia and that the sinking of this trough and its filling, mainly by the Yellow River silt, roughly keep pace with each other. However, it is possible that the enormous silt loads which yearly are being deposited on the North China alluvial plain and its coastal fringe tend to help depress the geosynclinic trough and thus further promote the thrust of the western mountain masses. Furthermore, if the sinking of the plain and the deposition of silt on it keep pace rhythmically, there may be periods when the coastal fringe of the delta will suffer an invasion by the sea, and other periods when it will be built up. These are interesting speculations and have bearings on the regulation of the river through the plain.

The Yellow River is peculiar in the way it encircles a part of its own area. Its most important tributary, the Wei Ho, which drains Central Shensi and Eastern Kansu, forms the "heart" of the river. It is bordered on the west, north, and east by its parent river. This has the effect of modifying the flood flow of the entire Yellow River as floods from the most up-stream part of the area seldom coincide with floods from the Wei Ho. This characteristic is further modified by the flow loss along the 900-mile course from Lanchow to the Suiyuan-Shansi border, where the river is merely a conveyance channel. However, the heavy flood flow which the river can receive on its ways from Suiyuan through the 400-mile course down to Tungkuan, at times, is likely to synchronize with the flood flow coming from the Wei Ho. Here is to be found one of the causes for the magnitude of the flood of the Yellow River which can be so destructive in the plain. The other cause is found in the floods of the tributaries—Lo Ho and Chin Ho, in Honan—which also can synchronize with the floods from up stream. This is due to the fact that the general path of many of the flood-producing rainstorms is from west toward east, thus following the flood crest down stream. The result, therefore, is the peculiar phenomenon that one-third of the entire drainage area up stream from Paotou, in Suiyuan, cannot contribute any more than 100 000 cu ft per sec, or one-eighth to one-tenth of the river's estimated maximum flood flow of 800 000 to 1 000 000 cu ft per sec where it enters the plain. As regards the silt flow, a similar situation exists. Where the river leaves Suiyuan and enters Shansi, the maximum silt percentage that the river has been observed to carry rarely reaches 5% by weight; generally, it runs less than 1 per cent. Contrast this with more than 40% at Lungmen, which lies at the lower end of the gorge between Shansi and Shensi, and nearly the same value for the Wei Ho where it enters the Yellow River. These findings regarding the distribution of the river's flood and silt flow are some of the most important contributions to the knowledge of the hydraulics of the Yellow River during the seven years since 1930.

It should be mentioned that the Wei Ho (drainage area, 56 000 sq miles) is almost as important a flood and silt carrier as the Yellow River itself, al-

[7] "The Polar Control Theory of Earth Development," by A. W. Grabau, *Journal*, Assoc. of Chinese and American Engrs., May–June, 1937.

though it is less important as a low-water feeder. Its most important tributaries are the King Ho (23 000 sq miles), draining mainly East and Northeast Kansu, the Lo Ho (10 000 sq miles), draining North Shensi, and the Hu Lu Ho (5 000 sq miles), draining Southeast Kansu. These three streams all enter the Wei Ho from the north. As the loess and tertiary clay areas lie mostly to the north of the Wei Ho, it follows that the northern tributaries are the silty ones whereas those coming from the Chin Ling Mountain Range to the south may almost be classed as clear rivers, in comparison. The silt coming from the Wei Ho area is, in general, much finer than that coming from the areas in Shansi and Shensi between Hochu and Tungkuan. Partly for this reason freshets from the Wei Ho have a tendency to scour the diked channel of the Yellow River, whereas those from the Hochu-Tungkuan area, containing much coarser silt, tend to raise the river bed. In general, however, freshets from the Wei Ho area have more variation in the intensity of their silt loads than those from the Hochu-Tungkuan area which are always heavily silt-laden.

Hydrological Considerations

Meteorology.—North China has two distinct yearly climatic seasons—the dry late fall, winter, and spring season with its northerly monsoons caused by the Siberian high-pressure area; and the rainy summer and early fall season with its southerly monsoons caused by the low-pressure area over interior China. In the northern regions of North China the rainy season tends to begin later and end earlier than in the regions closer to Central China. As the course of the Yellow River traverses most of the territory which may be termed North China, its low-water season is shorter and its high-water season longer than the corresponding seasons of the smaller rivers in the north, such as those belonging to the Hai Ho System which converge on Tientsin. The flood season of the Yellow River, therefore, is reckoned from the middle of June to the end of October; but usually it falls between the end of June and the end of September. There is also a brief high-water period in the beginning of April due to release of the ice cover in Suiyuan and North Shansi. From Tungkuan to the sea the ice has already left the river by the end of February, but, where it flows through Suiyuan which lies 5° to 6° farther north, the ice cover does not break up until late March or the beginning of April. At this time ice jams usually form along the flat, sloping course of the river through Suiyuan. These jams may have considerable storage of water behind them. Frequently small freshets from the region above Lanchow occur at the same time. Due to its more southerly location, this section thaws out a little earlier than the river in Suiyuan. Together they form the early April freshets. It should be noted that along the diked part of the river through the alluvial plain a somewhat similar situation exists. The course near the mouth lies near Latitude 38°, whereas the up-stream diked course lies near Latitude 35°; hence, this part thaws out first and causes ice jams in the river near the mouth. These ice jams may be serious enough to break the dikes, as in late February, 1928.

Central China and North China get their rainfall from two systems of cyclonic disturbances, the continental system of low barometric pressure areas

which originates in the interior of Asia and which drifts eastward into the Pacific Ocean, and the Pacific Ocean system of depressions or typhoons which generally originates east of the Philippine Islands and which drifts either westward toward South China or curves north and northeastward following the coast of Asia and then back into the Pacific Ocean over the Japanese Islands. These two systems serve to bring into violent contact the dry, cooler air masses of the North Asia Continent and the moist equatorial and South China ocean air masses causing condensation and rainfall. The constant presence of dust in the continental air masses may accentuate the condensation. The south monsoons serve to drive the moist equatorial and Indo-China air masses northward and thus make moisture available for condensation when the depressions arrive on the scene. During the winter the "high" Siberian pressure area, with its strong anti-cyclonic northerly winds, drives moisture southward. This factor, combined with low temperatures, makes the available moisture for condensation and precipitation much less when the depressions arrive from the continent than during the summer, which explains the dry winters and springs although the depressions are numerous enough. Often these depressions merely cause dust storms. When there are no depressions hovering about, there is a gradual tapering off of one air mass into the other. The weather is then generally fine and settled all over China and the typical monsoonal winds blow steadily.

It should be noted that the continental depressions drift against the sources of available moisture in the Pacific Ocean, and, therefore, the rainstorms set up by them are usually short-lived; in the winter and early spring they often cause little or no precipitation. During the summer, however, their drift may be slow, and they may remain stationary for some days causing prolonged, copious rainfall. In order to cause rain on the Yellow River area, the Pacific cyclones must pass close to, or inside, the China coast. Such paths seem possible only during July, August, and early September, and hence typhoons cause flood-producing rain during this period only. The paths of all the depressions are determined by the indistinct contact line of the equatorial—Indo-China and the continental air masses, between which the depressions tend to pass. The primary cause for the rainfall is thus the two opposing air masses; and the secondary cause is the depressions that serve to bring them in contact in such a way that condensation occurs.

To forecast floods in the Yellow River from meteorological observations only seems feasible and may become quite reliable after more study has been made of the various combinations that produce severe rainfall. Considerable study has already been made by the various observatories in China to unravel the rain-storm mysteries; but meteorological stations are lacking in West China and in the regions still more to the west. This has prevented students from forming reliable conclusions as to what climatic combinations will produce the most serious floods in the Yellow River. For the present, flood warnings are sent from "key" hydrometric stations equipped with wireless sending apparatus. This is fairly satisfactory and will generally give 24 to 48 hr advance notice of floods before they arrive at the upper section of the diked course. However, the floods from the Lo Ho and the Chin Ho (which enter the river

near the head of the diked channel) constitute a serious difficulty. In July, 1935, the flow at Shanchow was reported telegraphically as 250 000 cu ft per sec, whereas at the hydrometric station just below the Peiping-Hankow Railway Bridge it was measured and reported as 586 000 cu ft per sec. The warnings from the Lo Ho and the Chin Ho came too late and, mainly because the river guards were taken unaware, a destructive breach occurred in Western Shantung. Such flow synchronism, only not so severe, also occurred in the summer of 1937. The flood of July, 1935, was due to a continental depression. A still larger flood of 635 000 cu ft per sec occurred in August, 1935, at Shanchow, traced to typhonic causes. It had increased to only 653 000 cu ft per sec at the head of the diked section.

Little information is available regarding rainfall measurements on the Yellow River drainage area prior to the organization of the Yellow River Commission in 1933. One of the first steps this Commission took was to establish a rainfall-measuring service and, as far as possible, it enlisted the cooperation of the Provincial Governments. Before the flood season of 1934, rainfall was measured at more than 250 stations scattered mostly over the area down stream from Suiyuan, but including the Wei Ho area. Rain-gages, 8 in. in diameter, patterned after the United States Weather Bureau standard, were installed, and it has been possible, from the data collected during 1934 and 1935, to compare rainfall and run-off. The rainfall values have been computed from 1-in. isohyetal maps by the use of planimeters. As the rainfall stations were too few up stream from Paotou in Suiyuan to draw the equal rainfall lines accurately, only the area down stream from this hydrometric station was used and the flow coming from up stream deducted.

The results show a remarkably small run-off percentage. The rainfall from August 1 to 10, 1937, resulting in the high-water period from August 1 to 18, gave a ratio, run-off-to-rainfall, of 13.8 per cent. The rainfall period from August 1 to 10, 1935, as compared with the run-off period from August 1 to 17, had a ratio of 9.2%, and the rainfall period of July 1 to 10, 1935, with a run-off period from July 7 to 13, gave a ratio of 16.1 per cent. The latter storm covered only the Wei Ho area and the Yellow River area down stream from Tungkuan. The flow observed at the Lungmen station and that of the Fen Ho coming from Shansi entering below the Lungmen station were deducted. These data show, conclusively, the effective absorbing power of the loess areas and contrast greatly with similar observations for many of the tributaries to the Hai Ho which drain the more loess-denuded eastern slopes of the Mountain districts immediately west of the plain. Run-off percentages as high as 75% have been observed on these loess areas. Unquestionably, many of the smaller mountain tributaries of the Yellow River can show high run-off rates; but, taken as a whole, the Yellow River has a low run-off rate, which is one of the main reasons why this river, having such a large drainage area, does not yield a higher flood flow than 900 000 cu ft per sec. The Yangtzekiang, up stream from Ichang, with a somewhat similar drainage area, has been observed to have a discharge of nearly 2 500 000 cu ft per sec in a heavy flood.

Frequency and Magnitude of Floods.—Since it is the control of the floods in the plain that is most imperative, one naturally is concerned with the flood flow that enters the diked section of the Yellow River. Hence, the most useful records of flow would be those obtained close to the head of that section. Unfortunately, this flow has been observed only for a few years. The nearest are those for the Shanchow station where practically continuous records are available dating from late 1918. That station is 185 miles up stream from the head of the diked section, and the drainage area down stream is 12 000 sq miles. The latter has several highly important tributaries, such as the Lo Ho (5 000 sq miles) from the south and the Chin Ho (4 000 sq miles) from the north, both of which enter the main river practically where the dikes begin. Unquestionably, the Lo Ho can have flood flows as high as 350 000 cu ft per sec and the diked Chin Ho perhaps as much as 70 000 cu ft per sec. (An automatic control by overflowing the plain before its course has been diked precludes any higher flood-crest flow from entering the Yellow River.) Twice since 1934 the Lo Ho has been observed to have had freshets of 250 000 cu ft per sec—once in 1935 and, again, in 1937. These were exceptionally large floods, however.

It has previously been mentioned that the flow of these two rivers, especially that of the Lo Ho, may synchronize with the flood crest of the main river. Except the flow of these two rivers and other smaller ones belonging to the drainage area between Shanchow and the head of the diked section, however, the Shanchow data record the flow of the entire Yellow River area of 278 000 sq miles up stream from it.

The Lokou records from 1918 to 1937 contain the flow of the river after all losses (such as those from percolation, evaporation, and dike breaches) have occurred. Obviously, they are not very useful for developing a proper flood-magnitude frequency relation. In the past these records have been much referred to as representing the flow of the Yellow River.

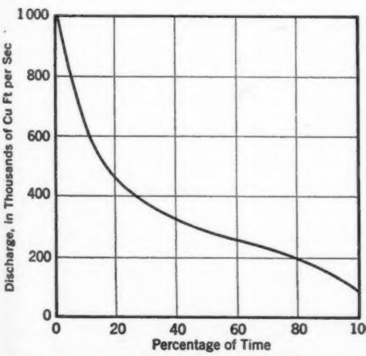


FIG. 16.—FREQUENCY RELATION; YEARLY MAXIMUM FLOODS

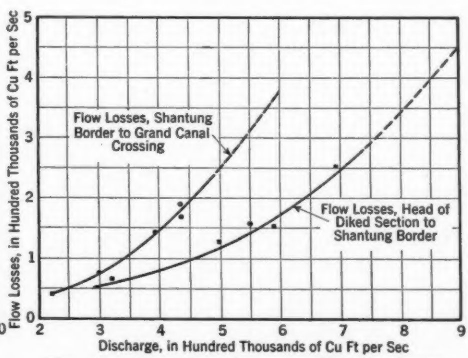


FIG. 17.—FLOW LOSS DIAGRAM, YELLOW RIVER

The duration curves for maximum annual floods have been drawn in Fig. 16, using the 19-yr flow records at Shanchow (1918–1937). It is obvious that, at this point, the Yellow River can have a flood flow exceeding that of 1933,

which has been interpolated to have been approximately 812 000 cu ft per sec. Even in 1933 this value could have been exceeded if a closer coincidence had occurred between the freshet of the Wei Ho and the freshet coming from the main river up stream from Tungkuan. The floods are very flashy both from the Wei Ho and from the main river; therefore, a few hours' difference in the arrival of their flood crests at the confluence point at Tungkuan will make a decided difference in the magnitude of the resulting floods passing down the main channel below Tungkuan.

Usually, when the main river has a freshet, there is none, or a very small one, from the Wei Ho—or *vice versa.* As far as has been observed it is the rainfall produced by typhonic causes that tends to bring about simultaneous freshets in both systems. Furthermore, serious floods from typhonic causes are less frequent than those from continental depressions, which generally produce freshets in only one of the systems; but such freshets are again likely to synchronize with the Lo Ho and Chin Ho floods. When floods are produced by typhonic causes, however, the Lo Ho and Chin Ho floods run off first. An extreme flood at Shanchow caused by typhonic influences, therefore, is not likely to be seriously increased by run-off from the area down stream, such as those produced by continental depressions.

As far as can be judged the river is not very likely to have a flow exceeding 900 000 cu ft per sec at the Peiping-Hankow Railway Bridge. To provide for floods greater than this seems economically unsound. This expected maximum flood is about 100% greater than that estimated by earlier investigators. In a long-range forecast of possible maximum floods, perhaps, one should also take into consideration the probability of undertaking drainage-area improvement, such as erosion-control measures. If it proves feasible to reduce erosion to an extent that will decrease the silt content of the river in heavy floods, from 40% to approximately 10%, this alone will reduce a flood of 900 000 cu ft per sec to about 700 000 cu ft per sec by silt removal, not to mention the ameliorating effect which the improvement works themselves may have in retarding the surface run-off and in assisting the soil absorption. Considering the question carefully it seems that 900 000 cu ft per sec is a conservative value to adopt in future design for flood-control measures.

TABLE 2.—MAXIMUM ANNUAL FLOOD DISCHARGES AT SHANCHOW, IN CUBIC FEET PER SECOND

Year	Discharge	Year	Discharge	Year	Discharge	Year	Discharge, at:	
							Gage	Head of diked section
(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(3)
1919	459 000*	1924	152 000	1929	399 000	1933	812 000	825 000†
1920	247 000	1925	459 000*	1930	247 000	1934	399 000	512 000*
1921	353 000*	1926	254 000	1931	265 000	1935	635 000	685 000†
1922	247 000	1927	212 000	1932	371 000	1936	477 000	547 000
1923	371 000	1928	159 000	....	....	1937	424 000	459 000*

* Dike breach. † Serious dike breach.

The maximum yearly flood discharge of the Yellow River, at Shanchow, for the years 1919 to 1937, inclusive, is shown in Table 2.

Flow Losses.—A matter of much importance for the regulation of the Yellow River is the loss by soil absorption, percolation, and channel storage, along the diked course. These losses are so important that they will figure definitely in the design for the capacity of a regulated channel. The river flows on a ridge with the water level higher, generally, than the plain on both sides; and, the plain itself slopes away from the river toward both sides, which further promotes percolation. Due to the constant dike breaches it is somewhat difficult to reach definite conclusions; but, from the available records some opinion may be formed. Such losses should be observed for various conditions of flow because evidently a high but flashy freshet, lasting scarcely more than 24 hr, is certain to suffer greater losses than another freshet with longer duration. Furthermore, if the river's foreshores are saturated by a recent flood, they will absorb less water than if the foreshores were dry. A silty freshet may or may not have heavier losses than a freshet that is less silt-laden. Heavy rainfall on the wide area between the dikes when a freshet passes down may partly offset losses. These are some of the many factors having a bearing on this question. The loss due to channel storage is a temporary one and its effect is to flatten the flood crest as it proceeds down stream. It is a detention effect only, without loss in volume.

Considering only the diked course from Lokou to Litsin near the mouth, a distance of 110 miles, the average monthly low-water loss in April, May, and June, 1935 (months when the flow was not affected by dike breaches) was 62, 82, and 54 cu ft per sec per mile of river, respectively. As the flow during these months averaged, at Lokou, 39 148, 39 783, and 43 525 cu ft per sec, and, at Litsin, 32 405, 30 886, and 37 736 cu ft per sec, the losses ranged from 13% to 22% of the flow at Lokou, which correspond to average losses in ordinary irrigation channels. Considering the elevated position of the Yellow River with reference to the plain the loss may be regarded as small. Due to complications of dike breaches in the upper course of the diked river and also due to seepage flow from the Shantung Mountains it has not been possible to arrive at any valuable results from that part of the river. The bed is much wider at that part and it is probable that the low-water losses are heavier than for the lower river.

During the flood season it is only to be expected that the flow losses will be relatively greater than during the low-water season. The wide foreshores become inundated and since, as a rule, they slope from the river banks toward the dikes, and usually are well cultivated, this feature will facilitate percolation losses. Much silt is deposited on the foreshores and for every cubic foot of settled silt about 0.7 cu ft of water will be held with the silt. For example, if there is a loss in silt discharge of 10 000 cu ft per sec (dry state) between two points on the diked river it means that, at the same time, 7 000 cu ft per sec of clean water is tied up with the deposited silt. Although small, this loss is a permanent one and may be regarded as part of the percolation losses.

On account of the wide distance between the dikes, the flattening out of flood crests by channel storage is always most marked from near the head of the diked course at the Peiping-Hankow Railway Bridge to the border of Shantung, a distance of 135 miles. As the dike distance narrows it becomes

less marked along the next 70 miles to the Grand Canal crossing. (When there are dike breaches up stream, as in 1934, there will be a return flow in the vicinity of the Grand Canal crossing and this obscures the actual amount of loss.) Along the down-stream 185 miles, from the point where the return flow enters to the last hydrometric station at Litsin near the coast, the channel is much narrower and the average distance between the dikes is only slightly more than 1.25 miles against an average width of 4.5 to 5 miles up stream from the Grand Canal crossing. Along this relatively narrow channel, judging by the observed flow data and allowing for observation inaccuracies, the decreases in cresting flow due to channel storage will usually not be great. The lengthening and flattening of the flood crest by up-stream channel storage has already become so marked that any possibility for further modification is practically eliminated. In fact, due to the flattening of the slope, which is quite pronounced up stream from Litsin, there may be a slight increase of the cresting flow at times, due to the overtaking effect. Observations tend to show that this may take place. From these considerations it is evident that the point for determining what the down-stream channel capacity should be under regulated conditions lies somewhere near the Grand Canal crossing because, from this point down stream, there will be a tendency for the flow to remain relatively unaltered. The loss or gain that may occur along the course from the Grand Canal crossing to the sea does not seem to be large enough to warrant any alteration in channel section. The observed losses during flood periods in 1934, 1935, and 1936 have been plotted in Fig. 17.

Assuming that the losses are proportional to channel area some conclusions may be reached as to what the losses will be in a regulated channel of much smaller dimensions than the present unregulated channel. To convey a flow of 900 000 cu ft per sec the average regulated width between the new dike lines will be about 2.5 miles for the upper reaches of the river from the Peiping-Hankow Railway Bridge to the Shantung border and 2 miles from this place to the Grand Canal crossing. The present dike distances for these two stretches are about 6 miles and 4 miles, respectively. The diagram for the upper stretch, using a flood of 900 000 cu ft per sec at the Peiping-Hankow Railway Bridge, gives a flow loss of about 450 000 cu ft per sec and for the second stretch about 200 000 cu ft per sec. The proportional losses for the new channel would then be about 187 000 cu ft per sec and 100 000 cu ft per sec for the two stretches. For rainfall effect it would be conservative to allow only 150 000 and 80 000 cu ft per sec, or a total loss of 230 000 cu ft per sec between the Peiping-Hankow Railway Bridge and the Grand Canal crossing.

It was noted during the flood season of 1937 that prolonged, medium freshets of 300 000 to 350 000 cu ft per sec did not register as large losses as the more flashy freshets. This is to be expected, of course; it has a bearing on the regulated channel capacity for the condition when there are flood-control detention basins in the mountain area west of the plain. The maximum flow that is likely to enter the diked section in this case will probably not exceed 550 000 cu ft per sec, but it may last for more than 24 hr. Under this condition the flow loss between the Peiping-Hankow Railway Bridge and the Grand Canal crossing will probably not be more than 150 000 cu ft per sec and may

even be as small as 100 000 cu ft per sec. This will be sufficient control, however, to prevent a flow of more than 450 000 cu ft per sec from passing into the reach below the Grand Canal crossing.

The writers realize that further investigations and study are necessary to solve more conclusively the dependable magnitude of flow losses along the diked channel; but they are of the opinion, nevertheless, that the present observations show that these losses are so consistently present, and so large, that they should be taken into consideration in a control and regulation plan for the diked course. In fact, if they are not taken into account it will result in an over-sized down-stream channel which will be difficult to maintain. Further investigations may show that the values adopted tentatively by the writers are too small.

Fig. 18.—Gorge of the King Ho, Shensi, at Intake of Wei Pei Irrigation Project, Where Measurements of Samples Have Shown as High as 50% of Silt by Weight.

Silt Flow.—There is perhaps no important river in the world in which silt is so intimately related to every phase of conservancy work as the Yellow River. Whether it is flood control, irrigation, hydro-electric development, or navigation, the question as to how to overcome the evil effects of the silt overshadows all other technical considerations. This is due to the staggering proportions of the load carried. On other rivers a silt content of 1% or 2% by weight is considered a heavy and obnoxious charge which causes much trouble; but where the Yellow River enters the plain the silt load reaches 40% by weight at times. On several of its tributaries charges as high as 50% by weight have been observed (see Fig. 18). The silt percentage is expressed as: $\frac{\text{Weight of dried silt}}{\text{Weight of water silt}}$; whereas, the customary method is to express it as: $\frac{\text{Weight of dried silt}}{\text{Weight of filtered water}}$. The first ratio is used in order to avoid such absurd silt percentages as 80%, 90%, or, at times, even 100 per cent.

For computing volumetric silt quantities it has been assumed that the silt density is that of the dried silt ordinarily found on the plain, the weight of which averages 101.5 lb per cu ft (1 600 kg per cu m); that is, its specific gravity is 1.6. Furthermore, from observations based on numerous experimental samples, the average specific gravity of the individual silt grains has

been taken as 2.7. Using these values the relation between silt percentages and silt discharges can be developed. Fig. 19, which shows this relation, is used in preference to curves based on experimental data, due to the average condition which the computed curve represents.

Since a knowledge of quantities of silt is more useful than a knowledge of weights, the curve has been made to represent the volume of silt contained in a unit volume of river discharge corresponding to any observed percentage by weight, of silt, in the river discharge, assuming that the silt has been naturally deposited on the plain and has dried. The silt discharge is found by multiplying the number obtained in Fig. 19 by the observed discharge. This curve is very convenient for use when one has to convert silt percentage and discharge into silt discharge a considerable number of times. Most investigators use a conversion factor for computing silt discharge from silt percentages. This may be correct enough for very low silt charges; but, obviously, it gives incorrect results for higher silt percentages since the variation is not a straight-line relation.

Fig. 19 has been based on the assumption that the silt has been allowed to settle, drain, and dry naturally. Therefore, it should not be used for calculations involving silt deposits temporarily settled in flood-control detention basins. Silt settled in water has a volume from 5% to 20% greater than that which the curve gives and a density of 1.8 to 2.1, depending on the character of the silt, time of settling, and depth of the deposits. Loess silt and water become plastic when the percentage by weight reaches 56 to 58. Coarse silt, settled under water, attains a definite volume rather quickly, whereas fine silt continues shrinking for a long period. Hence, each silty stream on which flood detention basin control is contemplated must be especially investigated with regard to its silt-settling action.

In the Yellow River, as well as in other heavily silt-laden rivers of North China, it does not necessarily follow that a maximum flood will have the maximum silt concentration. It may have the maximum silt discharge, however. Several factors determine when the Yellow River will have its maximum silt concentration, some of which are: (*a*) Intensity of rainstorms and area covered by them; (*b*) place where intense rainstorms hit the drainage area; (*c*) dry or wet climatic cycles; (*d*) dilution effect from other parts of drainage area; and (*e*) heavy bed scour during serious freshets.

The most important factor in modifying the silt charge of the river's flow is perhaps the climatic cycle. The silt concentration in the flow has gradually diminished since 1934 and this is unquestionably due to the wetter cycle which began in 1931 and may continue for some years more. It has resulted in a better growth of bushes and grasses over the entire drainage area to such an extent that, during the dry winter and spring, the farmers have not been able to cut the bushes down or to scrape the grass and leaves away completely. This increased vegetation is now holding the soil more securely in place over areas large enough to have a marked influence upon both surface and gully erosion.

The bank erosion along the river channels has probably not been affected; but, in any case, bank erosion is a secondary factor among those which ordi-

narily contribute to the silt charge as it usually alternates with bank deposition all along the course. On the other hand, channel bed scour may be a very important factor in throwing large masses of bed silt into the stream. For example, in the reach up stream and down stream from Lungmen, in August, 1933, the river bed was deepened an average of more than 15 ft along a stretch estimated at 30 miles. It is unfortunate that no definite record of this extraordinary phenomenon exists other than that obtained from information after the flood. At Lungmen the bed was deepened approximately 30 ft over a width of 3 000 ft, tapering off in depth up stream and down stream. It has been estimated that a silt quantity approximating 7 000 000 000 cu ft was

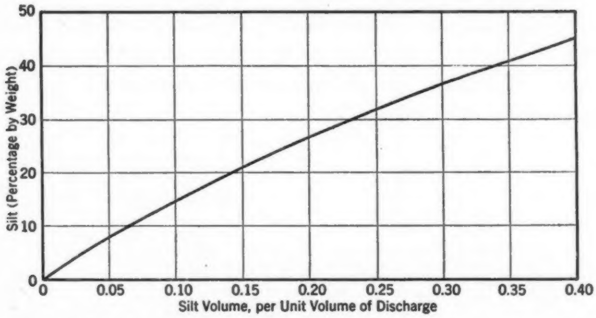


FIG. 19.—CONVERSION OF SILT PERCENTAGES TO SILT VOLUMES IN TERMS OF UNIT VOLUME OF DISCHARGE

eroded from the river bed along this stretch in a time which cannot have greatly exceeded 12 hr. It demonstrates the capacity of the river to scour in spite of the high silt load it was already carrying. The average discharge during this period seems to have been about 300 000 cu ft per sec. Since this occurrence in 1933 the river bed has again been built up more than 12 ft. A part of this enormous silt quantity was redeposited in the wide river channel up stream from Tungkuan; but, at Tungkuan, and probably down stream toward Shanchow, the bed was again lowered 3 or 4 ft. Similarly, in the plain up stream and down stream from the Peiping-Hankow Railway Bridge, the bed was scoured nearly 6 ft by the same flood that deepened it at Lungmen. At these places the bed remained depressed after the flood, but was partly refilled during the spring of 1935. It seems quite possible that this extra silt load was at least partly responsible for the deposition in the plain which buried the north dike and caused overtopping and numerous dike breaks in August, 1933, below the 1851 break.

However, such occurrences are infrequent. In fact, the inhabitants at Lungmen cannot remember that the river bed has ever been lowered so seriously before. They say: "The river lost its bed"; but old boat fastenings, and iron rings bolted to the rock and uncovered when the bed silt was removed, show that the normal river bed has once been deeper. This points to cycles of silty and less silty periods, which again means dry and wet periods. There is no other stretch along the river where such abnormal action can occur. Regulation of the Yellow River, from the gorge outlet at Lungmen to Tung-

kuan, with a flood-control detention basin up stream from Lungmen (as is advocated herein under the heading, "Flood Control and Regulation"), will remove, or at least minimize the effect of, such a flood as occurred in August, 1933.

Normally, however, the river receives its silt charge from surface and gully erosion of the loess lands in Kansu, Shensi, and Shansi, and to a lesser extent from Honan. The parts of the drainage area from which the river receives its silt are shown in Table 3.

TABLE 3.—Percentage of Silt Observed in the Yellow River

Month	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average
	(a) Paotou			(b) Lungmen			(c) Shanchow			(d) Railway Bridge*		
1934:												
January	....	....	....	....	....	....	0.35	0.30	0.32	0.44	0.24	0.32
February	....	....	....	....	....	....	0.56	0.29	0.40	0.81	0.36	0.49
March	....	....	....	....	....	....	1.22	0.40	0.82	1.50	0.49	0.74
April	....	....	....	....	....	....	3.15	0.62	1.13	1.90	0.52	0.91
May	....	....	....	....	....	....	2.30	0.69	1.22	1.60	0.74	0.98
June	....	....	....	....	....	....	3.05	1.02	1.75	2.81	0.58	1.38
July	2.55	0.17	0.89	38.00	1.98	8.20	20.16	2.37	6.86	9.42	1.14	5.46
August	2.35	0.88	1.48	36.22	1.70	6.36	38.14	3.03	9.01	18.75	2.78	7.88
September	2.13	1.08	1.47	6.38	0.97	2.35	8.14	2.23	3.77	6.84	2.02	3.58
October	1.96	0.99	1.38	2.57	0.89	1.52	7.62	2.77	4.04	8.15	1.73	4.03
November	....	....	....	1.52	0.43	0.82	2.71	1.39	2.13	3.46	1.13	1.99
December	....	....	....	0.70	0.07	0.27	1.85	1.20	1.62	1.77	0.67	1.19
1935:												
January	....	....	....	0.31	0.03	0.17	1.95	0.91	1.23	1.38	0.40	0.92
February	....	....	....	0.78	0.04	0.22	1.95	1.23	1.50	1.47	0.73	0.93
March	....	....	....	2.78	0.26	0.87	3.05	1.64	1.93	2.61	0.79	1.62
April	....	....	....	1.11	0.34	0.60	2.94	1.59	1.91	1.49	0.90	1.21
May	....	....	....	1.35	0.44	0.82	2.68	1.61	2.17	1.70	0.97	1.25
June	....	....	....	1.38	0.54	0.97	2.55	1.00	1.68	1.45	0.87	1.14
July	1.11	0.72	0.87	13.37	0.75	3.81	14.71	1.17	4.40	10.75	1.07	4.26
August	1.14	0.81	1.01	33.46	0.53	3.28	19.34	1.88	5.31	15.26	2.91	5.37
September	1.00	0.78	0.91	3.73	0.75	1.54	4.10	0.99	2.17	3.86	1.46	2.53
October	1.22	0.68	0.98	1.87	0.64	1.10	3.22	1.17	1.99	3.18	1.42	1.92
November	0.94	0.42	0.61	1.25	0.32	0.69	1.98	0.85	1.40	3.02	1.29	1.95
December	0.46	0.20	0.32	0.55	0.03	0.17	1.45	0.32	0.98	2.71	0.57	1.42
	(e) Kaotsun			(f) Taochengpu			(g) Lokou			(h) Litsin		
1934:												
January	....	....	....	....	....	....	0.25	0.07	0.14	....	....	....
February	....	....	....	....	....	....	2.05	0.06	0.40	....	....	....
March	....	....	....	....	....	....	0.79	0.19	0.44	....	....	....
April	....	....	....	....	....	....	2.93	0.55	1.08	....	....	....
May	1.12	0.51	0.70	2.06	0.82	1.72	2.21	0.61	1.30	....	....	....
June	2.33	0.79	1.40	3.03	0.63	1.30	4.07	0.50	1.83	....	....	....
July	6.25	0.87	3.93	7.52	0.90	4.23	6.35	0.58	3.66	5.53	0.52	3.41
August	11.42	2.54	5.58	12.26	3.22	6.24	10.97	2.35	5.69	10.92	2.47	6.43
September	5.43	1.40	3.28	5.65	1.89	3.10	5.86	2.34	3.51	5.43	1.73	2.29
October	5.12	1.74	3.21	3.47	1.69	2.53	4.15	1.68	2.66	5.36	2.10	2.92
November	2.87	0.63	1.75	3.41	1.14	1.95	2.12	0.68	1.26	2.29	0.52	1.27
December	1.74	0.41	0.65	2.13	0.09	0.46	1.00	0.20	0.40	0.63	0.17	0.35
1935:												
January	0.72	0.19	0.39	0.44	0.08	0.22	0.38	0.11	0.22	0.23	0.10	0.17
February	0.49	0.15	0.30	0.31	0.08	0.16	0.31	0.09	0.18	0.16	0.11	0.14
March	1.34	0.32	0.66	1.73	0.13	0.51	1.62	0.27	0.50	1.14	0.15	0.39
April	1.53	0.48	1.07	2.68	0.22	1.13	1.77	0.32	0.89	1.33	0.33	0.75
May	1.68	0.84	1.18	1.94	0.77	1.25	1.86	0.67	1.31	1.60	0.50	1.02
June	2.59	0.85	1.17	3.69	1.10	1.75	2.17	0.96	1.38	2.03	0.95	1.33
July	8.22	0.40	2.48	5.29	0.81	2.27	5.48	0.67	1.77	6.01	0.89	2.28
August	10.19	0.62	3.60	9.56	1.34	3.21	5.32	0.45	1.41	4.48	0.61	1.68
September	3.57	0.91	1.93	6.44	2.05	3.60	2.38	0.85	1.35	2.81	0.93	1.64
October	2.55	1.08	1.60	4.07	1.23	2.82	1.55	0.72	1.02	2.04	0.43	1.17
November	2.57	0.85	1.68	1.96	0.21	0.74	0.81	0.06	0.29	0.62	0.06	0.24
December	2.00	0.05	0.71	0.39	0.02	0.18	0.13	0.03	0.06	0.10	0.02	0.04

* Peiping-Hankow Railway Bridge.

When comparing the data in Table 3 it is necessary to consider the effect of the dike breach at Kuantai in 1934 and at Tungchuang in 1935. Although the Kuantai breach had no appreciable effect in reducing the silt concentration during the flood season at the stations down stream, it had much effect in reducing them during the low-water season that followed, on account of the clarified outflow from the flooded areas north of the river between the inner and outer dikes in Shantung. The Tungchuang breaches, on the other hand, caused considerable reduction in the silt concentration down stream immediately after the breaches had taken place, due to much deposition between Tungchuang and the Grand Canal crossing and to the clarified flow that re-entered the river at Chiangkow below the Grand Canal crossing.

The daily silt percentages during the freshet season in 1934 and 1935, as they were observed at the hydrometric stations from Shanchow down stream, are listed in Table 4. The sudden increases in silt concentration during freshets, and its gradual diminution and tapering off as the flow passes down stream, is very striking. Table 5 gives interesting silt records gathered in 1934 and 1935 on the Wei Ho.

It has been observed that the river is quite capable of conveying a 12% silt-laden flow of 200 000 cu ft per sec from the Grand Canal crossing to the sea, without any marked deterioration of the channel in that stretch, in spite of the fact that the slope is materially flatter than up stream. It did so during the August, 1934, freshet. The records of this flood are shown in Table 6.

As noted heretofore, the maximum silt concentration does not always occur at the same time as the maximum discharge. Usually, it comes some time after the crest has passed. Quite frequently, however, the maximum silt discharge occurs at the same time as the maximum stream flow. The silt percentages that coincided with the cresting flow are shown as Item No. 3, in Table 6. Actually, the discharge that conveyed the heaviest silt concentration down the diked channel was smaller than the cresting flow. The channel that had been scoured was refilled during the passage of the crest, but not so much as to leave it in any worse condition than during the freshet, except in the reach immediately down stream from the Kuantai breach. It is unfortunate that dike breaks have obscured, somewhat, the happenings along the middle reaches of the diked section. Otherwise, it would have been possible to interpret more accurately the normal action of the river, for instance, with regard to the scour and refill action between the hydrometric stations. All one can conclude for the present is that, at times, it seems deposition can occur in the channel between the hydrometric stations during freshets. Table 4 (*b*) indicates that this perhaps took place between Lokou and Litsin during the August, 1934, freshets, but not to any great extent. The recorded maximum discharge at Litsin at that time is probably too small, and this again would affect the silt discharge. The silt concentration in the flow shows no appreciable change.

The average monthly silt discharges for 1934 and 1935, as they have been computed for all the hydrometric stations along the river below Shanchow, are given in Table 7; the total yearly silt volumes passing the principal hydrometric stations along the Yellow River and the Wei Ho for the same two years, are given in Table 8.

TABLE 4.—CHANGES IN SILT CONCENTRATION ALONG THE COURSE OF THE YELLOW RIVER

(Percentage of Silt by Weight)

(a) OBSERVATIONS IN 1934

Date (1)	Shanchow (2)	Chinchang (3)	Kaotsun (4)	Taochengpu (5)	Lokou (6)	Litsin (7)
July 6	5.25	3.83	2.35	2.45	2.98	1.74
	4.30					
7	3.77	3.59	2.68	2.23	2.55	1.93
	5.35	3.89				
8	5.47	3.91	2.98	2.12	2.20	2.41
	9.47					
	13.14					
9	14.38	7.93	2.74	2.24	2.01	1.08
	14.93					
10	14.63	8.37	5.27	3.66	2.57	2.14
	13.28					
11	10.42	8.62	4.26	7.52	3.36	2.15
	9.02					
12	8.56	8.23	2.77	5.18	6.35	3.70
	8.39					
13	6.92	7.49	4.80	4.59	5.12	4.98
	5.09					
14	3.93	7.44	5.32	4.70	4.20	4.69
	5.01					
15	2.84	8.17	6.25	5.28	5.04	3.66
	3.40					
16	3.88	5.68	5.53	5.28	4.66	3.45
	4.73					
17	5.87	3.91	6.08	6.13	4.66	3.62
	5.98					
18	3.89	4.16	4.91	5.50	4.52	4.24
	10.99					
19	9.38	4.62	3.86	5.06	5.13	4.55
	11.60	5.44				
20	14.66	5.22	3.42	4.46	5.05	4.51
	20.16					
21	9.87	7.79	3.97	4.95	4.81	5.04
	8.78					
22	8.15	9.42	4.62	4.74	4.76	4.92
	9.54					
23	8.32	8.16	5.56	5.10	4.64	4.62
	7.32					
24	6.81	6.86	6.23	4.56	4.44	4.28
	6.45					
25	3.36	5.28	5.59	5.15	4.74	4.30
	5.11					
26	4.36	6.05	4.99	4.78	5.59	4.35
	7.71					
27	5.90	5.11	4.33	4.27	3.95	5.52
	5.79					
28	5.44	4.58	4.94	5.14	4.54	3.95
	4.21					
29	3.75	5.25	4.33	5.67	4.39	4.32
	4.21					
30	3.25	3.93	4.00	6.42	3.53	4.34
	3.03					
31	3.89	4.57	3.29	6.00	3.78	4.33
	2.45					
Aug. 1	3.29	3.39	2.88	3.22	3.02	3.72
	3.03					
2	4.50	2.78	3.33	3.39	3.04	3.57
	4.95					
	4.21					
3	6.96	3.98	2.54	4.03	2.43	3.05
	6.09					
4	6.36	4.78	3.08	....	2.35	2.47
	8.45					
5	7.80	7.10	3.30	4.35	3.10	2.75
6	6.23	9.55	3.72	4.54	3.91	3.04
	8.28		3.93			
7	7.71	8.89	4.96	4.13	3.71	4.01
			5.26			
8	6.96	6.22	5.30	5.40	3.61	4.33
	4.23		5.10			

(b) OBSERVATIONS IN 1935

Date (1)	Shanchow (2)	Chinchang (3)	Kaotsun (4)	Taochengpu (5)	Lokou (6)	Litsin (7)
July 6	2.44	1.47	1.48	....	....	....
	2.65	1.48	1.44			
7	2.48	....	1.39	....	0.95	1.07
	2.81		1.55		1.10	0.95
8	5.10	....	1.78	....	1.02	0.98
	4.91		2.46		1.18	1.02
9	4.31	5.62	4.12	....	1.20	....
	3.93	4.93	3.77		1.21	
10	3.41	4.22	1.61	....	2.06	1.60
	3.30	3.68	0.95		2.12	2.07
11	2.86	2.54	0.40	....	4.26	2.07
	1.98		0.57			5.01
12	3.10	2.36	0.80	....	5.48	4.79
	2.50	2.02	1.31		3.24	6.01
13	3.22	1.72	1.57	....	1.82	3.53
	4.01	2.38	2.74		1.53	4.69
14	5.40	2.53	3.32	....	1.29	2.39
	14.00	2.90	2.43		0.96	1.95
15	14.71	4.85	2.66	....	0.83	1.33
	11.27	5.25	2.44		0.74	1.43
16	11.84	....	4.24	....	0.76	1.41
	10.45	10.71	8.22		0.67	1.31
17	4.97	9.91	6.57	....	0.91	1.48
	5.35	10.41	6.64		1.56	1.57
18	5.06	8.47	6.41	....	1.79	1.78
	5.89		6.20			1.78
19	7.13	7.16	4.23	....	2.56	1.89
	10.44	7.58	5.22		2.33	2.47
20	7.72	6.93	4.17	....	2.60	3.57
	4.84	7.01	3.60		2.90	3.63
21	4.87	8.89	4.20	....	2.44	3.40
	3.38		4.46		2.85	4.01
22	4.00	7.16	3.60	....	3.29	3.50
	3.86	5.32	6.29		2.87	3.47
23	4.18	4.66	3.90	....	2.55	3.48
	3.83	3.93	4.51		3.04	3.90
24	3.02	3.79	3.41	....	2.36	3.15
	3.48	3.84	3.47		2.11	3.60
Aug. 3	2.43	4.39	5.47	2.25	1.50	1.64
	1.94	4.85	4.41	2.79	1.33	1.62
4	3.63	4.65	3.53	2.60	1.45	1.64
	4.43	3.59	3.78	2.84	1.36	1.44
5	5.87	3.22	1.80	3.80	1.52	1.62
	5.85	3.35		3.60	1.71	1.62
6	6.27	3.76	2.63	3.07	1.68	1.70
	3.71	3.58	3.59	2.45	1.63	2.13
	7.63					
7	11.65	6.21	2.87	2.92	1.71	2.07
	17.55	5.39	3.11	2.43	1.20	
8	19.20	6.02	1.35	2.32	1.16	1.89
	19.54	7.20		2.74	1.24	1.67
9	14.08	13.13	8.75	4.51	1.17	1.63
	10.10	12.90	3.77	5.57	1.30	1.77
10	9.98	13.41	10.19	6.10	1.94	1.50
	7.21	15.26	4.57	9.56	2.40	2.05
11	6.41	10.86	6.00	3.49	3.65	2.67
	5.15	11.53	5.76	3.49	4.32	3.26
12	3.97	7.64	6.36	3.08	2.66	3.73
	3.50	7.98	6.19	3.61	2.57	4.48
13	3.66	5.73	6.36	2.59	2.19	3.27
	4.12	5.86	8.09	2.32	2.16	3.13
14	3.81	4.07	4.11	2.28	1.43	2.14
	3.97		3.22	2.08	1.35	1.60
15	2.52	4.11	4.48	2.16	0.91	1.86
	1.98	4.25	2.79	2.05	1.01	2.42
16	2.41	4.54	3.26	1.81	0.98	2.11
	1.88	3.10	3.72	1.96	0.91	1.39
17	2.13	3.45	3.59	1.78	0.74	1.24
	2.08		2.84	1.90	0.87	1.66

TABLE 4.—*Continued*

Date (1)	Shan-chow (2)	Chin-chang (3)	Kao-tsun (4)	Tao-cheng-pu (5)	Lo-kou (6)	Lit-sin (7)
(a) Observations in 1934 (Continued)						
Aug. 9	20.90	7.55	4.80	5.15	4.69	4.46
	16.41		4.82			
10	18.64	10.41	6.07	6.25	6.14	5.30
			5.86			
11	31.82	16.65	7.13	5.54	4.96	5.10
			5.48			
12	21.21	15.22	8.95	10.22	5.14	5.14
	20.88		8.44			
13	24.21	15.77	6.56	12.26	7.88	5.31
	38.14					
14	16.11	18.75	4.19	9.61	8.66	8.35
			4.88			
15	10.66	15.89	7.87	8.83	8.42	10.70
	6.83		4.36			
16	5.52	10.55	3.89	10.67	7.74	9.40
	4.55		10.60			
17	5.88	6.97	11.42	9.65	10.97	9.58
	4.73		8.89			
18	3.17	6.75	9.19	8.83	10.11	10.92
	5.02		7.63			
19	5.56	6.49	5.56	6.41	7.67	9.46
	5.25		6.04			
20	3.57	4.59	5.18	6.42	7.15	9.07
	3.86		5.62			
21	11.52	2.82	5.20	5.96	6.94	9.00
	9.61		4.62			
22	8.16	4.84	4.30	4.51	5.03	9.00
	9.96	5.04	4.71			
23	7.44	7.18	5.89	6.29	5.62	9.03
	5.81		5.08			
24	4.63	8.09	4.66	6.56	6.17	9.00
	4.52	8.46	5.35			
25	6.82	6.05	6.59	4.79	4.90	8.84
	7.16	6.05	6.32			
26	11.48	5.86	5.89	6.03	6.23	8.73
	11.09		5.06			
27	8.60	7.32	5.28	4.51	6.33	8.55
	7.09	6.40	6.06			
28	6.15	9.34	5.77	5.77	4.90	6.07
	6.32	7.60	5.32			
29	6.66	5.76	6.36	5.62	5.05	5.14
	7.42		6.38			
30	5.83	5.10	5.46	5.36	5.59	5.46
	6.90	5.03	4.30			
31	5.13	5.82	4.04	4.08	5.86	5.06
	4.00		4.61			
Sept. 24	2.35	2.81	2.83	3.10	2.34	2.34
	2.31	3.98	2.67	2.54	2.86	
25	2.34	2.49	2.36	2.70	3.24	2.59
	2.56	2.04	2.01		2.85	
26	2.48	3.20	1.40	2.56	2.82	2.55
	2.23		1.90			
27	2.23	3.52	1.64	2.79	2.84	2.35
	2.23	3.52	2.03		2.77	
28	3.17	2.73	2.14	2.40	2.71	1.73
	3.13	2.57	1.98		2.95	
29	3.05	3.97	2.08	2.77	2.53	2.13
	3.14		1.66		2.47	
30	2.81	2.86	2.19	1.89	2.79	2.44
	2.37		2.31		2.70	
Oct. 1	2.80	3.42	2.25	2.92	2.74	2.40
	3.23	3.16	2.83		2.93	
2	4.81	3.12	4.85	3.07	2.71	2.30
	5.98	2.69	3.55		2.70	
3	7.87	6.85	2.85	2.91	2.94	3.11
	7.07	7.47	2.82		2.80	
4	7.52	4.98	4.19	3.21	2.83	2.80
	6.61	5.48	4.63		3.32	
5	5.93	8.15	4.62	3.47	3.58	2.69
	5.27		4.37		4.15	

Date (1)	Shan-chow (2)	Chin-chang (3)	Kao-tsun (4)	Tao-cheng-pu (5)	Lo-kou (6)	Lit-sin (7)
(b) Observations in 1935 (Continued)						
Aug. 18	1.95	3.48	2.27	1.73	0.63	1.13
	3.26	3.36		1.90	0.45	0.96
19	3.05	3.18	2.26	1.64	0.72	0.84
	2.46	3.47	1.94	1.34	0.71	0.78
20	2.31	3.14	2.16	1.44	0.59	0.79
	2.66	3.15		1.36	0.61	0.86
21	3.55	3.53	2.78	1.57	0.63	0.96
	3.15	3.39	2.28	1.59	0.54	0.66
22	2.90	3.36	2.51	2.42	0.65	0.61
	4.19	3.39	2.04	1.72	0.54	0.79
23	3.79	5.29	1.77	2.14	0.54	0.69
	3.77	5.66	1.86	1.94	0.76	0.67
24	4.65	2.99	2.81	2.50	0.64	0.67
	4.73	3.52		2.67	0.64	0.63
25	4.75	....	4.82	2.69	0.68	0.73
	5.83		3.28	2.16	0.77	0.89
26	6.01	2.91	2.22	5.56	1.09	0.87
	5.68	3.65	2.88	3.82	1.51	1.09
27	5.21	6.30	3.82	3.60	1.31	1.23
	5.08	4.48		3.60	1.71	1.26
28	3.63	6.94	3.52	6.89	2.01	1.54
	4.05	4.91	2.96	5.42	1.83	1.57
29	5.74	3.85	3.23	6.65	2.36	1.95
	6.84		2.31	5.75	2.26	2.29
30	7.88	3.74	2.99	5.13	2.11	2.52
	7.88	4.55	1.69	4.57	2.26	2.90
31	4.74	4.13	0.62	4.75	1.79	2.47
	3.43	3.70	2.37	4.18	1.98	2.83
(a) Observations in 1934 (Continued)						
Oct. 6	5.03	4.78	5.12	2.40	3.46	4.05
	3.89	5.88	4.51		3.11	
7	3.31	5.27	4.42	2.36	3.46	3.80
	3.22	4.37	4.18		3.47	
8	3.23	5.59	3.53	2.30	3.11	3.79
	3.86	3.72	3.77		2.91	
9	3.46	2.99	3.61	2.58	2.71	2.99
	3.31	2.48	3.96			
10	3.62	3.37	3.15	2.50	2.98	2.80
	3.25		3.03		3.12	
11	3.05	3.18	3.07	2.44	3.14	2.26
	2.95	3.01	3.22		3.24	
12	3.35	2.85	2.71	2.67	2.27	2.93
	3.31	3.97	2.83		2.39	
13	5.10	3.02	2.91	2.60	2.33	2.64
	3.78	3.53	1.74		2.56	
14	2.79	4.59	2.25	2.39	2.93	2.83
	2.77	5.58	2.38		2.87	
15	3.23	4.35	2.34	2.49	2.32	2.91
	3.28	2.86			2.46	
16	4.50	5.33	3.44	3.08	2.48	2.22
	5.84	5.22	3.83		2.61	
17	4.45	4.73	3.52	2.84	2.42	2.99
	3.48	5.24	3.93		2.65	
18	3.22	3.11	3.29	3.47	2.65	2.95
	3.30	4.22	2.65		2.56	
19	4.28	4.37	2.61	2.40	3.18	5.36
	4.34	4.69	3.34		3.11	
20	4.25	3.53	2.90	2.35	2.81	3.19
	5.27		2.70		2.67	
21	4.13	4.26	2.90	2.35	2.64	2.99
	3.79	4.20	2.49		2.38	
22	3.39	2.03	2.83	2.42	2.60	2.84
	3.79	1.79	2.59		2.56	
23	3.65	2.64	2.73	2.51	2.67	2.50
	3.67	3.42	2.51		2.91	

TABLE 5.—PERCENTAGES OF SILT RECORDED BY STATIONS ALONG THE WEI HO

Year	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
(a) TAIYIN												
1934:												
Maximum	....	....	....	....	....	41.32	41.76	30.21	5.99	4.37	0.29	0.13
Minimum	....	....	....	....	....	0.01	0.34	0.42	0.17	0.38	0.08	0.06
Average	....	....	....	....	....	6.52	8.83	7.13	1.60	2.07	0.15	0.09
1935:												
Maximum	0.10	0.34	0.89	2.26	22.35	30.48	36.76	33.46	5.68	0.42	0.15	0.09
Minimum	0.03	0.03	0.30	0.92	0.04	0.08	0.09	1.00	0.10	0.03	0.03	0.02
Average	0.07	0.16	0.59	0.73	1.95	5.53	9.97	10.26	0.98	0.14	0.07	0.04
(b) HSIENYANG												
1934:												
Maximum	0.07	0.07	0.21	0.59	12.83	30.20	32.10	26.80	2.36	6.33	0.20	0.11
Minimum	0.05	0.05	0.09	0.01	0.12	0.19	0.68	0.44	0.28	0.19	0.07	0.06
Average	0.52	0.76	1.63	0.58	2.39	4.10	8.11	7.46	0.99	1.78	0.14	0.08
1935:												
Maximum	0.09	0.10	0.44	0.91	3.08	9.87	26.77	32.30	3.83	0.25	0.24	0.10
Minimum	0.02	0.03	0.13	0.25	0.24	0.51	0.40	0.53	0.12	0.08	0.05	0.01
Average	0.05	0.05	0.20	0.42	0.67	1.95	4.89	7.06	0.84	0.16	1.04	0.05
(c) HUACHOW*												
1935:												
Maximum	....	....	0.38*	0.98	1.82	9.36	29.31	26.58	6.27	0.38	0.26	0.19
Minimum	....	....	0.17*	0.29	0.17	0.32	0.24	0.15	0.22	0.08	0.13	0.07
Average	....	....	0.30*	0.50	0.46	1.97	5.27	6.97	1.03	0.18	0.17	0.13

* This station was established in the middle of March, 1935; the March records include only the last half of the month.

The average flow volumes and silt volumes for the periods of the more important freshets occurring in 1934 and 1935 for the three stations Shanchow, Chinchang, and Kaotsun, have been listed in Table 9, in order to give some idea of quantities to be handled in the flood-control detention-basin projects. It will be seen that more than 10% of the flow volume can be silt. The Wei Ho (Huachow Station) measurements in Table 8 do not include the flow of the Pei Lo Ho, which enters the Wei Ho immediately up stream from the confluence

TABLE 6.—REDUCTION IN SILT FLOW, AUGUST, 1934, FLOOD, ALONG THE DIKED CHANNEL

Item No.	Description	Shanchow	Chinchang	Kaotsun	Taochengpu	Lokou	Litsin
1	Maximum stream flow, in cubic feet per second	399 000	494 000	300 000	184 000	191 000	138 000*
	Percentage of Silt:						
2	Maximum value	38.14	18.75	11.42	12.26	10.97	10.92
3	Value of time of maximum discharge	21.00	15.00	6.00	8.50	10.50	10.00
	Maximum Silt Discharge:						
4	In cubic feet per second	60 000	52 000	11 800	10 400	13 400	9 000*
5	Expressed as a percentage of maximum stream flow	15.0	10.5	3.9	5.6	7.0	6.5

* Discharge value is questionable due to poor hydrometric equipment.

TABLE 7.—AVERAGE MONTHLY SILT FLOW OF THE YELLOW RIVER, IN CUBIC FEET PER SECOND

Hydrometric station	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
(*a*) 1934 OBSERVATIONS												
Shanchow	30	51	127	230	201	391	1 831	9 362	2 227	3 662	880	259
Chinchang	30	56	111	218	208	322	1 564	9 143	2 983	3 726	866	231
Kaotsun*	10	40	60	200	159	365	1 098	4 464	2 481	2 261	627	93
Taochengpu*	14	125	65	280	196	267	1 208	5 420	2 395	2 151	747	105
Lokou	15	112	67	273	185	447	1 114	5 454	3 524	2 711	751	136
Litsin†	20	140	80	270	180	440	885	4 935	2 687	2 462	937	174
(*b*) 1935 OBSERVATIONS												
Shanchow	178	208	467	446	615	478	3 028	7 731	1 680	1 334	576	151
Chinchang	171	131	296	244	324	283	4 625	6 015	2 517	1 723	909	323
Kaotsun*	25	11	78	235	332	315	2 142	3 897	1 736	929	707	228
Taochengpu*	26	19	131	246	336	494	1 256	1 103	937	638	44	11
Lokou	30	25	128	239	376	415	1 101	950	570	307	61	4
Litsin†	47	33	109	156	207	298	1 529	1 093	690	352	36	5

* From January to April, inclusive, the flow is estimated.
† Estimated from January to June, inclusive.

point with the Yellow River. Furthermore, the silt flow of the Fen Ho, which enters the Yellow River from the east below Lungmen, is not shown in Table 8. The silt flow of these two rivers will practically account for the rather large discrepancy of 7 500 000 000 cu ft by which the Shanchow Station measurements exceed the sum of those of the Huachow and Lungmen Stations.

Scour and Refill of the Yellow River.—It has been observed that some rivers, flowing in alluvial beds of their own making, tend to scour their beds when the discharge is increasing and again refill them upon a decreasing discharge. In America the Lower Colorado River is an outstanding example of such a self-scouring river. On the other hand, the tendency of the Mississippi River and the Yangtzekiang seems to be to raise their beds when the flow is increasing (at least, at the crossings) and to scour the deposited material when the river is falling. The causes of this difference in behavior are not readily determinable, but the character, quantity, and flow characteristics of the bed material, and of silt in suspension, may be some of the factors.

TABLE 8.—ANNUAL SILT VOLUMES OF THE YELLOW RIVER AND WEI HO, IN CUBIC FEET

Hydrometric station: (1)	1934 (2)	1935 (3)	Hydrometric station: (1)	1934 (2)	1935 (3)
Paotou	7 413 000 000	7 095 300 000	Chinchang	51 722 000 000	46 656 890 000
Lungmen	29 387 000 000	26 409 100 000	Kaotsun	31 552 720 000*	28 220 590 000
Huachow (Wei Ho)	....	10 991 940 000	Taochengpu	34 372 580 000	13 879 890 000‡
Shanchow	51 263 550 000	44 911 350 000	Lokou	39 191 910 000	11 147 350 000
			Litsin	34 987 040 000†	12 083 430 000

* Breach at Kuantai, between Chinchang and Kaotsun.
† The measurements for Litsin are probably somewhat low; January to June, inclusive, in 1934, are estimated quantities (see Table 7).
‡ Breach at Tungchuan on July 10, 1935, between Kaotsun and Taochengpu.

TABLE 9.—COMPARISON OF SILT DISCHARGE DURING FLOODS

Date: From: (1)	Date: To: (2)	Time, in days (3)	Silt flow, in cubic feet (4)	Total stream flow, in cubic feet (5)	Silt flood, expressed as a percentage of total stream flow (6)
(a) SHANCHOW HYDROMETRIC STATION					
July 7, 1934	July 12, 1934	5.00	1 623 800 000	23 177 980 000	7.0
Aug. 9, 1934	Aug. 17, 1934	8.00	16 687 793 000	153 865 774 000	10.8
Oct. 1, 1934	Oct. 8, 1934	7.00	4 066 560 000	110 164 593 000	3.7
July 6, 1935	July 11, 1935	5.50	1 255 586 000	51 771 651 000	2.4
Aug. 6, 1935	Aug. 11, 1935	4.75	11 311 638 000	118 195 855 000	9.6
(b) CHINCHANG HYDROMETRIC STATION					
July 8, 1934	July 13, 1934	5.50	1 141 443 000	20 067 344 000	5.7
July 8, 1934	July 17, 1934	9.00	15 694 715 000	152 496 000 000	10.3
Oct. 1, 1934	Oct. 9, 1934	8.00	4 565 737 000	120 624 336 000	3.8
(c) CHUNGMOU HYDROMETRIC STATION					
July 7, 1935	July 13, 1935	5.50	2 970 195 000	100 342 368 000	3.0
Aug. 7, 1935	Aug. 15, 1935	8.00	7 622 170 000	140 448 816 000	5.4
(d) KAOTSUN HYDROMETRIC STATION					
July 9, 1934	July 13, 1934	4.25	657 921 000	21 380 504 000	3.1
Aug. 9, 1934	Aug. 19, 1934	9.75	5 746 840 000	132 671 520 000	4.3
Oct. 2, 1934	Oct. 10, 1934	8.00	2 654 560 000	97 597 440 000	2.7
July 8, 1935	July 15, 1935	6.50	1 355 520 000	127 840 997 000	1.1
Aug. 8, 1935	Aug. 18, 1935	9.50	5 841 444 000	170 490 528 000	3.4

The Yellow River and most of the other silt-bearing rivers of North China exhibit the same characteristics as the Colorado River, even to a greater extent. In some places the Yellow River has been observed to enlarge its net cross-section, during the freshet season, to more than three times the area it has during the low-water season. However, the question (among many others) arises as to whether this river will stop its scouring, or even begin to deposit its load, when an extremely severe, silty freshet occurs. It is a non-regulated stream without any fixed bed width, being extremely wide and braided in many sections and very narrow in others, and its slope through the diked section varies considerably. It is not easy to predict how it would behave if it were to be regulated and made to flow in a bed of fixed width. The phenomenon of scour and refill, its magnitude, characteristics, dependability under all conditions of silty freshets, and the relation this phenomenon has to the silt transportation, therefore, seem to be of sufficient importance to warrant an investigation as to how far it will influence any regulation plans.

In order to examine the scour and refill action of the river, use has been made of the collected hydrometric data in order to decide whether some conclusions could be drawn. In general, the data gathered to date (1938) show a rapid channel enlargement and deepening upon the rising stage of a freshet and a partial refill immediately after the cresting flow has passed and the flow is

receding. After the freshet season has passed there will be a further gradual refill and by the end of the year the bed will have more or less the same elevation position it had before the freshet season began in the middle or end of June or the beginning of July, as the case may be. However, the reach up stream and down stream from the 1851 break is an exception. Here the river bed has been rising definitely during the five years, 1932–1937. In other respects, the scour and refill action is less pronounced in the up-stream reaches than in the down-stream reaches, but local conditions seem to have a considerable bearing on the magnitude of the phenomenon.

When the writers were trying to interpret the data highly dubious cases arose due to lack of cross-section measurements taken just at the time when important changes occurred in the river bed. Due to the long distances between the stations it has been difficult to correlate the happenings at one station with the happenings at the next station; and questions regarding the influence of the character of the bed silt and suspended silt also arose. At times, erosion and refill of the river bed occurred in a manner that could not be explained from the magnitude of the freshets, changes in velocity, the accompanying silt charge, or the rapidity with which the river rose or fell. Furthermore, the condition in the delta seemed to have a strong influence on the elevation of the river bed in the lower reaches. When the delta channel was favorable, the adjacent up-stream channel was deep, and the data tended to show that this increased depth propelled itself up stream by back-cutting until the influence several months later was noted as far up stream as the Grand Canal crossing. This conclusion needs verification by a better correlation of observations, however. The necessary "machinery" for all the investigation work had been set up, but had to be canceled on account of the war.

Some studies have been made of the variation in the effect of silty discharges on scour and refill by plotting silt discharge against total discharge on logarithmic paper for the conditions: (*a*) When the river is scouring its beds; (*b*) when it remains unaltered; and (*c*) when it is silting up. The data, although falling somewhat scattered on the graph, nevertheless show distinct straight line logarithmic relations in all three cases. The lines tend to show that the up-stream reaches of the river at low-water stages are more efficient for conveying silt without channel deterioration than the lower reaches; whereas, for a discharge of more than 90 000 to 100 000 cu ft per sec, the lower reaches become more efficient. These are relatively deep and narrow whereas the upper reaches are shallow, wide, and often much braided.

Variation in slope along the diked course does not seem to have any very decided influence on the capacity of this river to transport its silt or to scour its bed. Perhaps these phenomena may be determined more by the character of the silt (in the bed and in suspension), by the shape of the cross-section, and by the rapidity with which the flood flow increases—that is, by the accelerating force of the flow. Furthermore, the rate of refill seems to depend to a great extent on the rapidity with which the discharge decreases: Rapid decrease means a rapid refill, and *vice versa*. The steady refill back to normal low-water condition in the autumn is probably due to a slow "geschiebe" or bed-load movement when material deposited up stream is gradually moved down stream.

There has not been a chance to observe the scour and refill action during an intense, highly silt-laden freshet; but as far as has been observed a medium-sized freshet, silt-laden to the extent of 18% in the up-stream reaches and 12% in the down-stream reaches, does not seem to have its scouring capacity decreased materially. Higher silt concentrations may show detrimental effects, however.

These are the general conditions. The entire aspect is highly complex, and any silt-transportation and bed-scour formula to cover such multitude of unknowns, does not seem of much practical use. It is realized that more accurate and more specialized data are required to clarify the scour and refill problem. Data should be secured simultaneously at many places along the river with the sole purpose of obtaining the needed information. The river scours and refills so rapidly during freshets that special observation equipment must be organized for such study, including that of rapid silt analysis. Furthermore, the places where the observations are taken should be selected judiciously and should be numerous enough to eliminate possible local effects.

For the time being all that the writers can say with regard to the scour and refill which occur along the diked course of the Yellow River is that such actions do take place at all the five hydrometric stations along this stretch. However, the hydrometric stations have been placed where the river is relatively narrow and stable; no data have been obtained from places where the river is wide and braided, or at typical "cross-overs" between bends. It is possible that at such places there is little or no scour. There may even be a slight fill during the rising stages of a freshet. The possibility of such happenings should not be ignored in the design for a regulated channel. The writers, therefore, advocate cautiousness when it comes to deciding upon widths of a regulated channel for the diked course of the Yellow River.

The laboratory experiments in Germany (reported herein under the heading, "Scope of Investigations: Studies in Recent Years"), although illuminating, are not sufficiently conclusive to warrant any other description than "further experiments."

The gradual refill, especially in the lower reaches, which has been observed to take place in the autumn after the freshet season and which causes the river bed to be brought back to where it was before the beginning of the freshet season, may or may not occur if the channel were to be regulated. The bed-load may then be moved along the bottom more evenly through the diked channel and not, as at present, in fitful stages, first by deposition in the wide up-stream bed and then in another stage down stream. In the reaches up stream and down stream from the 1851 break the river is still adjusting its bed and this also obscures what the actual action would be in the lower reaches if the entire river profile were based on the delta at the sea as the main depositing area for the river through the plain.

Flood Control and Regulation

Methods of Decreasing Flood Hazards.—As the problem of decreasing flood hazards along the Yellow River through the Great Plain is now analyzed it appears that the remedy should be far more extended than merely to construct

greatly strengthened dikes. The remedies that should be considered carefully are the following:

(1) Detention basins west of the Great Plain to arrest temporarily, and to "iron out," the cresting flow of maximum floods;

(2) Spillway flow into basins on both sides of, and near, the river in West Shantung;

(3) A by-pass channel following the general course of the Tu Hai Ho from the Grand Canal crossing to the sea north of the river;

(4) Earth dikes and their protective works;

(5) Channel regulation and fixing a selected course;

(6) Control of soil erosion in Shansi, Shensi, Kansu, and parts of Honan; and,

(7) Control of run-off by increasing the brush and forest cover in Shansi, Shensi, Kansu, and parts of Honan.

Detention Basins West of the Great Plain.—The writers have come to certain tentative conclusions as to the value of detention basins along the main Yellow River above the delta, on several of the feeders from Shensi and Shansi, as well as on the Lo Ho and Chin Ho that enter in Honan at the beginning of the plain. Actual preliminary studies of the gorges below Shanchow were made under the direction of one of the writers in 1935 and 1936. In a paper[8] dealing with flood-control detention basins for the Yellow River, it is suggested that basins in comparatively narrow, steep reaches of the main river are more likely to be successful than any that may be created on the tributaries, due to a more sustained flow in the main river to wash out silt accumulation immediately after the passage of a flood. The aforementioned tributaries, however, have sufficiently sustained flows after flood periods to cause rapid silt removal from the basins behind the dams. Such basins will function both to smooth the flood crests (which are so extreme that they endanger the diked channel through the plain) and to detain a part of the extreme flood silt load, distributing it over a long period.

Beginning 8 miles down stream from Shanchow at the Sanmen Rapids and continuing down stream for 75 miles, the river runs through a series of gorges mainly in limestone formations. Either at Sanmen, or at other points in the gorge (such as at Palihutung), dam sites can be found that would be suitable for use in establishing flood and silt-detention control works. Above the Sanmen Rapids, 200 miles by river, are the Hu-kou Falls where another suitable site for a detention-dam reservoir has been found. This region was inspected by one of the writers and surveyed under his direction in connection with hydro-electric power investigations in 1934. The preliminary studies made of the Yellow River at this point and also those made of the Fen Ho in Shansi, 85 miles north of Taiyuanfu, at Hsiachingyiu,[9] as well as those made by the writers on the Chin Ho, in Shansi, and the King Ho and the Wei Ho, in Shensi, indicate that reasonably satisfactory sites are to be had for an extensive system of

[8] "Possibility of Yellow River Flood Control by Means of Detention Basins," by S. Eliassen, *Journal*, Assoc. of Chinese and American Engrs., July-August, 1936.

[9] "A Study of Shansi Rivers," by O. J. Todd, *Journal*, Assoc. of Chinese and American Engrs., January, 1934; also, "Shansi Water and Power Problems," by O. J. Todd, *Journal*, Assoc. of Chinese and American Engrs., July-August, 1935.

detention basins to retard flood discharge. Topographical maps indicate that equally good reservoir and dam sites are to be found in the upper waters of the Lo Ho, in Honan Province; but, thus far, no reliable reports have been made which would definitely show their worth for this purpose of river regulation. The general field has been investigated sufficiently, however, to encourage the writers to believe that the detention-basin feature of flood control of the Yellow River is feasible. Nevertheless, the silt difficulties are fully appreciated.

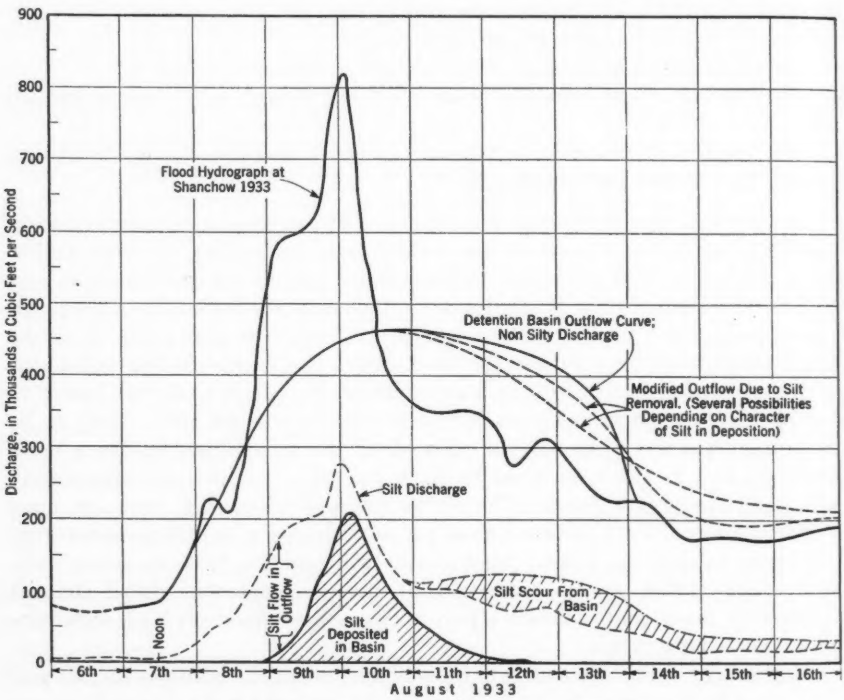


FIG. 20.—SILT DEPOSITION AND OUTFLOW FROM A DETENTION BASIN

Students of this problem generally assume that the openings in dams and canyon walls used for flow control must be ample so that the deposition of silt will not be too heavy in the reservoirs and that the deposits will be removed by scour. This limits the degree of control possible. Due to the complicated nature of the detention action, when there is much silt in the flow, it is not possible to compute, accurately, the data for establishing an outflow curve after the maximum stage has been reached in a basin. That the variations in the outflow and silt-clearing action may be many, can be seen from the diagrammatic sketches, Figs. 20 and 21.

If the basins are selected in relatively narrow valleys, with a slope steeper than 1 : 2 000, so that the normal velocity always will be a scouring one, there need be no apprehension that the basins will not clean themselves; nor is it possible that the large outflow openings may become choked. The important

questions are the time element involved in the silt clearing and the height of dam necessary. The time question has a bearing on the possibility of a second large flood following closely on the heels of the first; and the dam question is both one of general security and of economics. If the diagrammatic sketches in

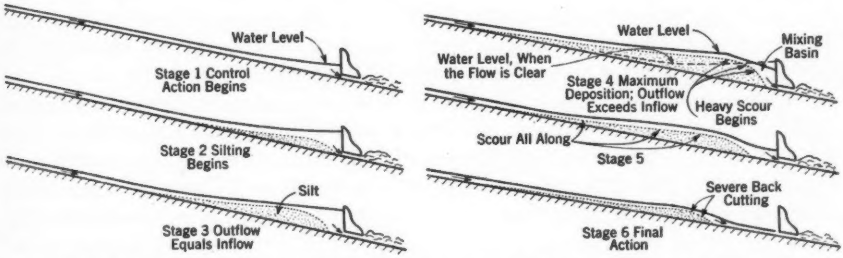


FIG. 21.—BASIN FLOOD CONTROL ON THE YELLOW RIVER, SHOWING PROBABLE SILT-DETENTION ACTION

Fig. 21 are studied, it will be readily seen that the silt deposition is likely to create a back-water situation which will increase enormously the storage facilities. The chances are that it will not be advantageous to have an over-sized dam in order to give added storage. There is room for much useful experimentation which, perhaps, could be best performed in some small tributary valley. The possibilities for controlling the Yellow River floods and silt by a system of detention basins are so promising that it is highly advisable to spend a considerable sum of money to clear up, by experimentation, such questions as cannot be solved by theoretical hydraulics or hydrometric observations of the river's flow characteristics.

The question of the effect of desilting major detention basins, such as the one proposed for the lower gorge on the main river immediately west of the Great Plain, in raising the river bed of the diked channel, has been considered and it is believed that records already gathered indicate an ability of the wide river bed between the Peiping-Hankow Railway and Mengtsin to hold, temporarily, such silt loads as are likely to be dropped above that point until the medium flow of the river can carry them out to sea through the autumn and winter months. This section of the river is nearly 75 miles long, is bounded on the south by high loess hills, and only for the last 8 to 10 miles on the north bank are there dikes, which may be raised when necessary. These can easily be protected against any temporary tendency to overtopping due to rapid bed-fill.

The ability of the Yellow River to pick up additional load and degrade its bed even in low and medium stages has been noted when certain local fills, as long as 50 miles, have existed due to new depositions that interfered with the flow unduly. The rapid cutting out and carrying to the sea of the silt filling the channel below the 1935 breach at Tungchuang, Shantung, was observed in late March and early April, 1936, as the closure of the breaches was completed and the river's flow of 30 000 to 35 000 cu ft per sec was forced through the old channel which had been choked with silt left there in the latter part of 1935. As it came into the plain in April (nearly 2% of silt by weight), the spring

freshet was not so heavily charged as to prevent its picking up, in the course of a week, the greater part of the fill in this section of the river channel. Furthermore, in the autumn of 1937, a sustained, medium-high flow of 150 000 to 200 000 cu ft per sec, silt-laden to 3% or 4% by weight, had a most potent influence in clearing out bed deposits that had been accumulating for one or two years in long stretches of the river below the Peiping-Hankow Railway. The main river channel is now as free from silt in its upper diked reaches as it was immediately after the series of high-water flows in 1933.

An examination of feasible detention-basin sites indicates that these basins could be relied upon to reduce a maximum crest flow by a little more than one-third (that is, detain a maximum flood of 900 000 cu ft per sec) so that it will not exceed 550 000 cu ft per sec at the Peiping-Hankow Railway Bridge. This discharge would be further reduced by channel losses to approximately 500 000 cu ft per sec at the Shantung border, and would arrive at the Grand Canal crossing with a flow of not more than 450 000 cu ft per sec, which can be carried by the down-stream channel without further control measures. By the adoption of this plan, it may not be necessary to resort to such other expediencies for flow reduction as are suggested in the following text, namely, overflow and desilting basins on both sides of the river in Shantung, and the by-pass channel following the course of the Tu Hai Ho from the Grand Canal to the sea.

Spillway Basins in Western Shantung.—In the control plan for guiding safely a maximum flood of 900 000 cu ft per sec through the regulated, diked section, it has been proposed, as one of the safety measures, to relieve the flow of 100 000 cu ft per sec by means of two spillways, one each on the north and south inner dikes in West Shantung immediately down stream from Tungchuang. If detention basins in the up-stream mountain areas prove feasible these spillways may not be necessary.

However, due to the fact that the river's section near Tungchuang is one of the most dangerous along the diked course under present conditions, and also because one must reckon with the fact that it will take considerable time before a large-scale improvement plan can be put into operation, since so many points remain to be investigated, it would seem highly advisable to construct such spillways at an early date in order to give added insurance against flood disasters during the period of preparation for the greater flood-control program.

The overflow from the spillways would pass into the areas between the inner and outer dikes, where it would be detained briefly and partly desilted, and then allowed to re-enter the main river just up stream from the Grand Canal crossing after the flood crest has passed the outflow points. Control works to regulate both the inflow and outflow would be necessary. As protection against the attacks of a prolonged flood, it may also be found advisable to have a set of escape sluices in the outer south dike just up stream of the outflow point in order to pass some of the flow into Tungping Lake from which it again would return to the main river at Chiangkou. Additional diking to limit the Tungping Lake area would be necessary. Likewise, there must be a strengthening of the south outer dike. The north outer dike (or the Golden Dike) has already been strengthened in view of the frequent dike breaks that have occurred on the north bank in late years, such as those at Fenglo and Kuantai.

The regions to be used for this purpose are low lying, are amply large, and lend themselves well to a part control of an excessive flow. The only obstacle is the number of villages involved. However, with an improved inner dike system it is not likely that these overflow basins would be used more than once in a decade; nor would the volume of flow into them be very large. The danger of their becoming rapidly filled with silt, therefore, would not be great. Each flooding would be during the summer only and the farmers would thus always be assured of a good spring crop. Often the flooding would not be severe enough to injure the summer kaoliang crop, except in the immediate vicinity of the spillways.

The villages within the basins would either have to be removed or raised. Such raised villages are very common in areas in the North China plain which are subject to frequent flooding and could be promoted with State support. The opposition to be overcome, however, will remain a very difficult part of this control measure. The farmers will submit themselves patiently to frequent chance disasters, but do not wish to listen to any controlled flooding although this may occur less frequently and with less destruction to themselves.

By-Pass Channel from Grand Canal to the Sea.—The Yellow River has no such natural by-pass channel as the Mississippi possesses in the Atchafalaya River. The nearest to it is a small stream called the Tu Hai Ho beginning near the Grand Canal north of the Yellow River and flowing to the sea to the northeast in a course nearly parallel to the main river. It has been suggested that this small stream may be excavated so that it can handle a flow of 120 000 cu ft per sec in times of high flood. Surveys of this project have been made by the Yellow River Commission and advance cost estimates have been developed. The writers have studied the data and feel that the cost of the proposed excavations is somewhat excessive compared to the benefits gained. However, two lines of substantial dikes may be built so that this stream-bed may be used as an overflow channel with a maximum width of about one mile. A suitable modern gate structure would be required near the Grand Canal crossing of the Yellow River to pass flood water from the main river into this new channel. To permit it to function properly it may even be necessary to put a regulator across the main river in order to throw the clearer low-water flow into the by-pass channel when necessary to scour away these silt deposits left during floods.

In case further studies tend to discourage the construction of large detention basins in the mountain districts west of the plain so that floods, after being reduced by channel storage and seepage losses, cannot be held to a maximum of less than 450 000 cu ft per sec at the Grand Canal crossing, it may be found advisable that this by-pass channel should be provided to take a flow of as much as 120 000 cu ft per sec. If it is assumed that a maximum flood will be nearly 900 000 cu ft per sec as it enters the diked section of the river without any modification by detention basins above that point, it may be estimated that not more than 150 000 cu ft per sec will be "ironed out" and lost by channel storage and seepage in the 135-mile stretch to the Shantung border. From a crest flow at that point of 750 000 cu ft per sec, approximately 100 000 cu ft per sec may be diverted to the two adjoining overflow basins in Western Shantung, and a further 80 000 cu ft per sec lost by channel storage and seepage in the next

70 miles. This means that a flow not exceeding 570 000 cu ft per sec will reach the Grand Canal crossing. Here, the proposed by-pass channel will start functioning in the case of an extreme flow. Such a by-pass channel will help in distributing the silt deposits at the coast over a wider area and thus retard the rapid delta building at the mouth of the river.

However, if detention basins in the main river near Shanchow, above Yumenkou, or at Hu-kou, and on some of the larger feeders in Honan, Shansi, or Shensi, prove feasible as aids in reducing crest flows, this by-pass may not be necessary. Assuming that detention basins plus flow losses reduce a maximum flood to 450 000 cu ft per sec by the time it reaches the Grand Canal, the present channel of the main river below that point can accommodate that quantity without any great improvement to that part of the system as it is.

Earth Dikes and Their Protection.—Although dike building has been practiced by the Chinese for more than 1 000 yr, they are to-day behind Western countries in construction methods and safety requirements. On the Yellow River the present earth dikes are not of uniform quality, slope, or width over the 430 miles that they extend from the beginning of the delta plain in Honan west of the Peiping-Hankow Railway to the tidal flats 20 miles east of Litsin; nor are they as scrupulously maintained as they should be, considering the highly important function they perform. Formidable dikes exist for the entire stretch along the north bank; but along the south bank from the Grand Canal crossing to the Tientsin-Pukow Railway, a distance of 90 miles, and from the Peiping-Hankow Railway westward, hills take the place of dikes. On the north bank the dike extends west from the Peiping-Hankow Railway for a distance of about 15 miles.

These dikes are made of loess soil, selected (if the supervision is good) from non-sandy areas and hammered down while wet in 1-ft layers by the use of a 90-lb stone, thrown 8 ft into the air by ropes managed by eight men, the flat surface of such stone being approximately 1 sq ft. By this means the dike is made reasonably tight, and voids that would permit serious leaks are generally prevented. Often the workmen are not carefully supervised, the layers become as thick as 2 ft, and the tamping is indifferently done. Furthermore, sandy soil may often be used with a thin surface layer of clay both for water-proofing and against wind erosion. Leaks through such dikes are difficult to stop and the result is often disastrous. When dikes are reconstructed in winter after dike breaks, frozen chunks appear which cannot be properly pulverized by the light tamping methods, and breaks have occurred from this cause. For these reasons the supervision should be made far stricter than it is at present. No dike construction should be allowed during freezing weather, and such construction can easily be avoided by a better use of the time when the freezing weather has passed. As long as the present light tamping methods are used for compacting the fills, the layers should never be more than 9 in. thick. It would add greatly to the efficiency of the dikes, however, if mechanical compacting devices, such as the American sheep-foot roller, were introduced. When the time comes for regulating the diked channel, the fill must be compacted mechanically or strict rules for construction methods must be enforced if disasters

are to be avoided. The new dikes will have to withstand water pressures and swift currents fully as severe as those to which the present dikes are subjected.

As a rule, the main dikes of the Yellow River are 25 to 30 ft wide on top, although some stretches built during the two decades (1918–1938) are as wide as 50 ft. Such excessive widths are partly due to poor, sandy soil, and partly to the fact that in times of high water it may be necessary to utilize the earth from the back of the dikes for making quick repairs. At such times the land outside the dikes is often flooded, due either to rainfall or to seepage, and it is costly to get earth from any other place than the dike itself. Spoil-banks of earth are also frequently placed on top of the dikes for such emergency conditions. These piles of earth which the Chinese call *tu niu* ("earth oxen") are familiar sights to travelers along the dikes. In all new dike work it would be well to follow this practice since, in the flood season, the elevation of the channel above the plain tends to create a swampy or flooded condition along the foot of the dike on the land side.

The present dike slopes are usually from 1 on 2.5 to 1 on 3.5, no uniform rule being followed as to which side should be the flatter. Generally, the dike slopes are covered with an indifferent growth of grass which seldom develops a first-class sod because, during the winter, the farmers rake the surface to gather the wilted grass for fuel and permit their sheep to graze on the dike slopes. Old regulations forbidding such practices are no longer enforced. During times of dike breaches people from the flooded areas crowd on top of the dikes where they build temporary shelters. After the breaches have been closed it is frequently impossible to persuade all of them to leave. Even when the dikes are raised these huts remain, gradually becoming caves in the back slopes with the roofs in some cases even lower than the top of the dike. The desire to live on the dike exemplifies the insecurity of the region along the river.

In recent years an effort has been made to plant willows on the foreshores close to the dikes. This should be encouraged as much as possible because such trees protect the dikes both from wave action and from the high-water currents. Five or more staggered rows of willows with the inner row 20 ft from the foot of the dikes and successive rows 10 to 15 ft apart afford an excellent protection. In the past, trees have been planted on the dikes and this practice still continues. As the dikes frequently have to be raised the trees become a nuisance and have to be removed. The roots remain in the body of the dike and are thus a source of leaks. However, the silty water tends to seal the finer interstices and generally tightens the dikes in the course of time so that this form of trouble is not so serious as it would be if the water were clear. Nevertheless, the potentialities for leaks are there and trees on dikes should not be encouraged.

Slipping of dikes due to excessive saturation is seldom experienced along this river although it does occur on the Yangtzekiang dikes where the clay content is often large and the difference between high and low water is as much as 50 ft at times; on the Lower Yellow River this difference seldom exceeds 15 ft. Dike failure may sometimes be caused by burrowing animals; but less apprehension need be felt for this possibility on the Yellow River than on the Han River near the Yangtze, for instance, where badger-like animals dig holes a foot or more in diameter. Great care should be exercised in watching for these holes.

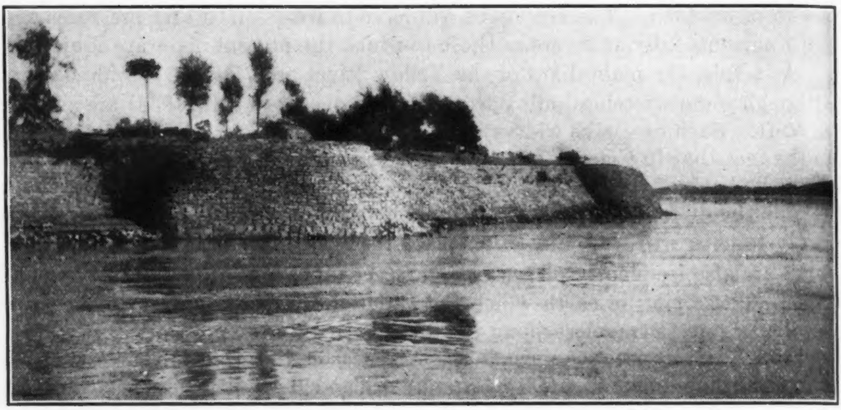


FIG. 22.—STONE GROIN BANK PROTECTION ALONG YELLOW RIVER IN SHANTUNG

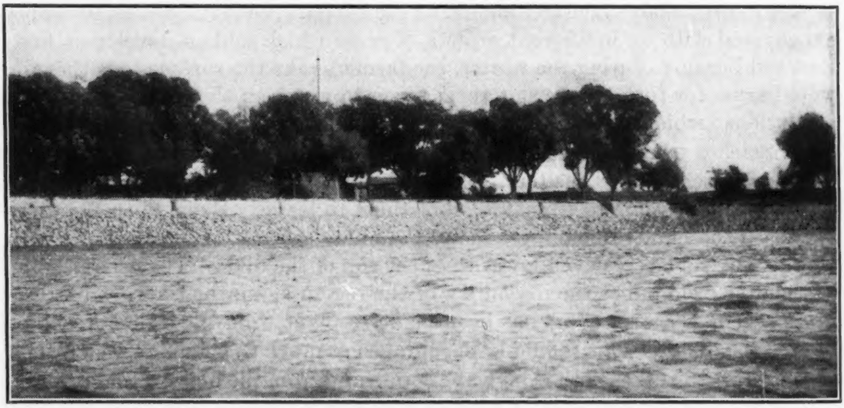


FIG. 23.—GOOD STONE WALL BANK PROTECTION WITH RIP-RAP ALONG YELLOW RIVER BANKS, JUST WEST OF TIENTSIN-PUKOW RAILWAY

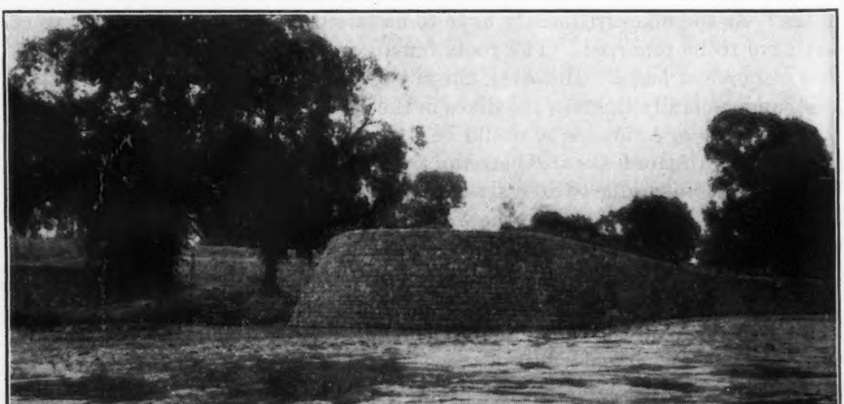


FIG. 24.—WELL BUILT STONE-PROTECTED GROIN POINTING DOWN STREAM ALONG SOUTH BANK OF YELLOW RIVER, 20 MILES BELOW LOKOU

FIG. 25.—NEW KAOLIANG BANK PROTECTION TIED BACK BY HEMP ROPES, ALONG YELLOW RIVER, IN SHANTUNG

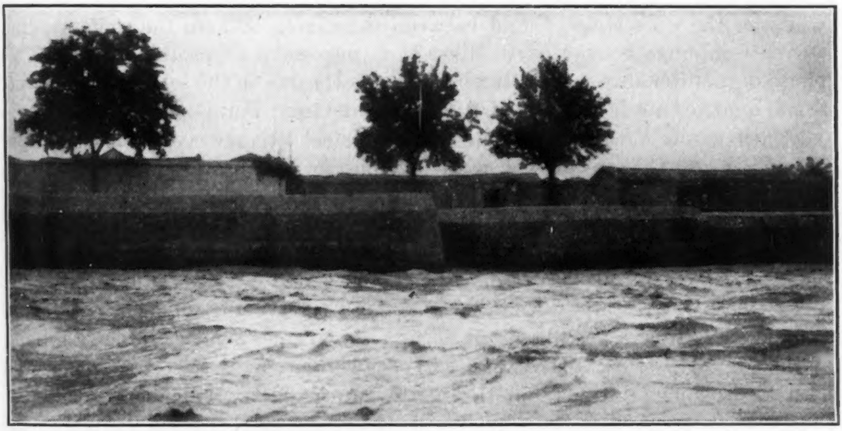


FIG. 26.—NEW KAOLIANG LAYER WITH EARTH COVER ON TOP OF OLD FISH-SCALE BANK PROTECTION ON YELLOW RIVER DIKE OPPOSITE LITSIN, SHANTUNG

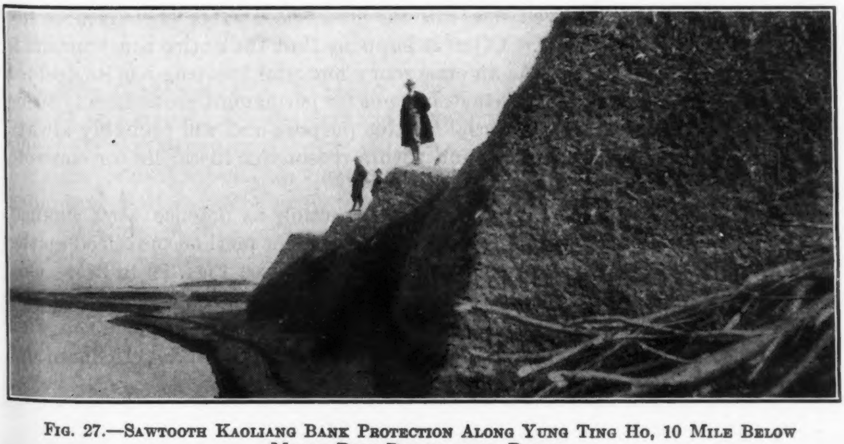


FIG. 27.—SAWTOOTH KAOLIANG BANK PROTECTION ALONG YUNG TING HO, 10 MILE BELOW MARCO POLO BRIDGE, NEAR PEKING

As the main dike system is frequently a mile or more from the low-water channel the foreshores are inhabited and cultivated, and the farmers living there have gradually thrown up small dikes along the edges of the channel to protect their villages and growing crops against flooding from medium high waters. In times of greater floods these dikes are overtopped and breached, and, as the main dikes are not protected, except where the current constantly impinges on them, the sudden onrush of water has been known, as at Tung-chuang in 1935, to set up highly eroding eddy currents which destroy the main dikes. Unauthorized dike work inside the main diking system may thus result in appalling disasters, and, therefore, in future regulation plans, human habitation must be prohibited inside the main diking system and any cultivation strictly supervised. With no villages on the foreshore lands, every one will be interested in keeping the main diking system intact, whereas, at present, there is a considerable number of people who often hope that the main diking system may be breached. The population living on the foreshore lands constitutes one of the difficulties in maintaining the river in its diked course.

Dike Protection.—Since the river channel is not stabilized it meanders at will over the wide strip of land between the dikes. Where the swift eroding currents impinge on the earth dikes it is necessary to protect them. This phase of maintenance work often becomes costly due to the long haul of stone. Stone quarries are few and far between in the Great Plain of North China.

Until a few decades ago when foreign steel became available for rapid quarrying, the Chinese used very little stone for dike protection works. The kaoliang stalk was the chief material used before that time and the kaoliang groin and "fish scale" pack work of the same material, so well described by Mr. Freeman,[3] are still used to a certain extent for dike protection in regions between the Shantung border and the Grand Canal crossing, and also on the lower river near Litsin; but in other regions they are being replaced rapidly by stonework, although for dike-break closure and in emergency cases the kaoliang stalk (Fig. 5) is still extensively used everywhere. Huge stacks of it may be seen to-day piled up near all the maintenance offices, which are spaced at intervals of about 20 miles along the river. Due to the great quantities required kaoliang is not a cheap material to use, and it needs constant maintenance because it rots out in two or three years. New work is then placed on top of the old, forcing the decayed mass into the mud below water where it is readily scoured into by the current. Often it happens that the entire construction is undermined and torn out. As an emergency material kaoliang has its decided place in maintaining the dikes intact, but not for permanent protection. Stone has now become the logical material for this purpose and will probably always remain so since sand cannot be found within reasonable distances for concrete revetment construction on a large scale.

Everywhere along the river, groin construction as defense work against swift currents is preferred to the continuous revetment method practiced on the Mississippi River. Many types of groins are used (see Figs. 19 to 27). One may see the triangular, pointed groin, the narrow, down-stream-pointing groin, and groins rounded in form. They consist of barricades of earth paved with stone and are exactly long enough to keep the current off the dike. In Shantung

they are mostly of the narrow type, the paved sides being almost vertical. A liberal depth of loosely thrown rip-rap covering of one-man-sized stone, protects the groin from under-water attack by the current. All groins are built to be above high-water level. Often an attempt is made to guide the current away from the dike by using one long groin; but this usually has resulted in causing added sinuosity of the river channel with more protection required for the opposite dike which becomes attacked. The "cure-all" effect of the single long groin is firmly defended by the old school of Chinese river engineers which discusses confidently what the river's action will be if a long groin were to be built in order to control a difficult reach. The reason for this optimism is probably due to the fact that the "trick" sometimes has worked without undue harm to down-stream parts. A decentralized control and lack of funds other than for immediate, pressing protection have prevented them from attacking, systematically, reach after reach and thus gradually subduing the river.

Tree retards as an emergency protection (see Fig. 28) are not greatly used along the Yellow River, although one of the writers experimented with this

FIG. 28.—USING TREE RETARDS DURING THE 1925 FLOOD IN WESTERN SHANTUNG TO SAVE ERODING BANKS AND DIKES ALONG THE YELLOW RIVER

type of bank protection in Shantung in 1922. Trees were used by the Chinese in 1925 at Linpuchi in an attempt to save an earth dike from further destruction. On the Yungting Ho, near Peiping, their use is much more common and has there proved valuable in emergencies. This remedy might be applied more commonly than it has been along the Yellow River.

Regulation of Diked Channel Through the Plain.—Closely related to the problem of maintaining dikes is that of regulating the course of the Yellow River through its diked section. In 1922, Mr. Freeman[3] proposed that there should be a straighter, narrower, and better protected course between two main dike lines than had existed theretofore. His plan was to have two dikes not

more than one-half mile apart and protected by regularly spaced groins. It was also his intention to have an inner channel, 1 800 ft wide, regulated and held in a fixed course so that it would be self-scouring and would readily carry all flood flows. Professor Franzius, of Hanover, Germany, using the data collected by Mr. Freeman, formulated an alternative plan in 1931, and recommended a slightly wider central channel, having inner unprotected dikes built along a theoretical mean-water channel expecting to retain the flow long enough in such a course as to make it scour its channel deeper. Each year these dikes would be repaired, but would not be protected in any way. This was a device to obtain a deep, narrow channel at low cost along which permanent dikes with suitable protection might eventually be built, thus stabilizing the channel. These eminent engineers both believed that a narrow channel with dikes built closely along the banks would make the river perform the work of maintaining a deep channel and that it would also scour sufficiently to accommodate the floods.

The hydraulic data that have been collected along the Yellow River since these two engineers discussed its channel regulation have thrown a much better light on the perplexities and difficulties which are inherent in the problem of making a stable, diked channel through the plain. In discussing this regulation problem, two alternative plans present themselves: (1) When the diked channel is to convey the modified, detained outflow from a system of flood-control detention basins within the mountain area to the west of the plain; and (2) when no detention-basin system is feasible and the diked channel must accommodate the full flood flow of the river.

The diked course is not without importance from the viewpoint of navigation and this should be given a certain weight in fixing the dimensions of the regulated channel. Let it be stated at once, however, that protection against flood disasters outweighs so greatly all other considerations that the question of navigation improvement must be considered after the flood-control measures have been fully met.

Under the heading, "Hydrological Considerations: Frequency and Magnitude of Floods," it has been stated herein that the writers consider a flood of 900 000 cu ft per sec to be the probable maximum discharge that the Yellow River can have at the head of its diked course. With a flood-control detention basin system west of the plain it may not be possible, due to the silt load, to reduce this discharge to less than 550 000 cu ft per sec where it enters the diked river. Allowing for channel storage and percolation, and other losses, this flow will most likely not exceed 450 000 cu ft per sec at the Grand Canal crossing, assuming the channel to be regulated. Below this point the present channel, with a moderate degree of improvement, should have sufficient capacity to convey this flow. For a prolonged period during the summer of 1937 it did convey almost this quantity, but finally breached the south dike near the coast due to insufficient bank protection.

Without a detention-basin system, however, the full flow of 900 000 cu ft per sec would have to be accommodated. This flow, it is estimated, would be reduced to 810 000 cu ft per sec at the 1851 break, to 750 000 cu ft per sec at the Shantung border, to 650 000 cu ft per sec by spillways into the proposed

overflow basins between the inner and outer dikes on both sides of the river between the Shantung border and the Grand Canal crossing, and, finally, by channel losses to 570 000 cu ft per sec at the Grand Canal crossing. At the latter station the Tu Hai Ho by-pass channel would take away a further flow of 120 000 cu ft per sec, allowing a flow of 450 000 cu ft per sec to pass down the present main channel of the river to the sea. Fig. 29 shows this

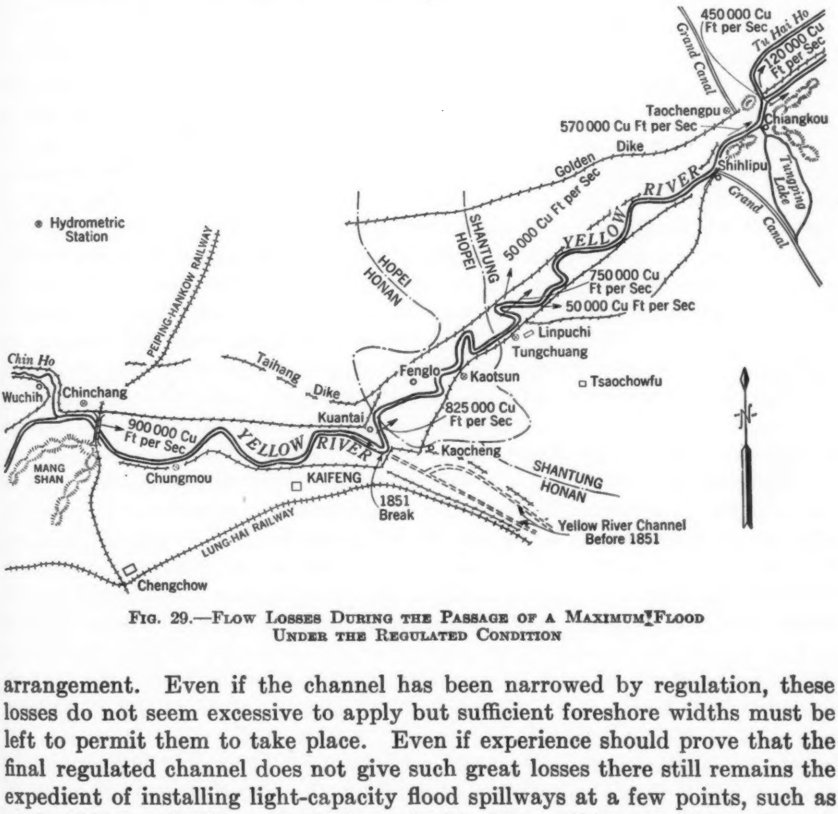


FIG. 29.—FLOW LOSSES DURING THE PASSAGE OF A MAXIMUM FLOOD UNDER THE REGULATED CONDITION

arrangement. Even if the channel has been narrowed by regulation, these losses do not seem excessive to apply but sufficient foreshore widths must be left to permit them to take place. Even if experience should prove that the final regulated channel does not give such great losses there still remains the expedient of installing light-capacity flood spillways at a few points, such as at the 1851 break, where the old channel still exists, and also below Tungchuang, where the proposed spillway over-flow basins would function for a long period before they became filled with silt.

As regards the distance between the dikes and the width and shape of the actual channel itself conveying the full flow of 900 000 cu ft per sec, there is room for much theoretical discussion. From the considerations of scour and refill, flow and silt losses, and also from the standpoint of the hydraulic laboratory experiments conducted in Germany, it is not at all certain that a narrow channel with dikes built closely along the banks will be the best type to adopt. The German experiments are especially illuminating; they tend to show, in a qualitative or indicative manner, that a channel with dikes spaced far apart scours a little better and silts up its foreshores somewhat faster than

the same channel with dikes built closely along the banks. This seems the logical result to be expected. By the transverse circulation of the flow a part of the silt load is transferred from the channel to the wide foreshores giving the less silty flow a better chance to pick up silt from the bed. If there are no foreshores the river has no other chance to relieve itself of its excess silt than by desilting it in the bed of the channel. However, even having a channel with wide foreshores there is certain to come a time when these have been silted up so much that the silt transfer has been reduced to a point where it no longer can influence the flow in picking up silt from the bed of the river. From this time, both the river bed and the foreshores will rise together. Taking a long view of the situation this is perhaps the condition which favors the slowest rise of the river bed. In selecting the width of the foreshores, therefore, it is of little use to make them wider than that required for a normal deposition width. The cross-sections of the river often show a break in the ground profile indicating the line where the heaviest deposition ends. This varies somewhat along the river, but a careful inspection of maps and cross-sections will point out, in a general way, the line to be followed for the best position of the dikes. At times, bad soil condition will make it impossible to follow this line closely, and the best practical route will have to be followed considering every aspect. Judging from cross-sections the indications are that foreshore widths need not exceed 1.25 miles, and, perferably, should be less than this on account of the cost involved in connecting a channel protective system to the dike lines.

There are also other reasons why excessive widths between dikes should be avoided. An outstanding example of detrimental, excessive width is the section between the 1851 break and the Shantung border. At the 1851 break the channel is more than 12 miles wide, whereas at the Shantung border it has been narrowed to about 5 miles. The consequence has been that the banks along the channel, due to the rapid silt deposition, have become as high as the top of the far-away dikes, as indicated by cross-section lines, and differences of more than 10 ft in elevation between the banks and the foot of the dikes have been noted. Generally, the transverse slope of the deposition is from 1 : 3 000 to 1 : 4 000 and hence the flow during freshets, when the foreshores overflow, runs diagonally across the foreshores cutting numerous channels which unite along the foot of the dikes, thus creating a very dangerous situation. It is almost impossible to stop the flow through these side channels once they have developed, and, in time, the river takes up a new channel along the foot of the dike. This has occurred time and again and when it happens the higher flood levels previously existing along the central channel are transferred to the dikes, and overtopping and breaching inevitably occur; hence, the necessity for both a narrower channel and a fixed channel as a link in the flood-control program. With foreshores not exceeding 6 000 ft in width the difference in elevation between the banks along the central channel and the foot of the dikes will be about 2 ft, which is not too much; but practical considerations may reduce this width to about 4 500 ft. The writers do not consider a width less than this to be advisable in the up-stream stretches of the diked river where most of the losses and silt deposition on the foreshores

will occur. It may gradually be reduced to 3 500 ft at the Grand Canal crossing. Below the Grand Canal crossing the dikes are not too far apart. They are even too close together in places. The main considerations here would be the straightening out of the existing dike system so that the entire length of the channel would have a more regular dike distance better co-ordinated to the channel itself in order to create a uniform flood profile with easier run-off conditions.

As regards the width to be given the actual channel, Messrs. Freeman and Franzius proposed 1 800 and 2 400 ft, respectively, as most practicable. However, they did not have available data regarding the actual flood flow or silt content of the river, which have been found to be more than twice the values they proposed to adopt.

When the river has a truly silty spell of 40% by weight as was measured during the flood of August, 1933 (812 000 cu ft per sec), there may be some doubt whether it will scour its bed even if this is regulated. The heavy silting which occurred below the 1851 break should be a warning. When designing the regulated channel width, therefore, no absolute reliance can be placed on a hope that the channel will enlarge itself. It seems better to be conservative and give it a width, at least in the up-stream reaches, which will carry the flow without making allowance for scour. The existing channel in the upper reaches of the diked section has widths varying from less than one mile to nearly two miles. At low water this channel is full of sand-bars with many breaches. As the narrower parts are fairly long, and since they carried the maximum flow in 1933 quite effectively, it would seem that under no circumstances should the regulated channel be wider than the narrowest existing sections. From an inspection of the actual conditions it would seem that it would be about right to give the proposed regulated channel a width not exceeding 4 000 ft in the upper reaches, gradually reducing this width to 3 750 ft at the 1851 break, to 3 500 ft at the Shantung border, and to 3 000 ft at the Grand Canal crossing. After the reduction of flow through the Tu Hai Ho by-pass channel a width of 2 000 to 2 200 ft may be sufficient along the entire distance to the sea. This is about the average width of the present channel. It is quite true that the width now at many sections is much narrower than this (as, for example, along the 3-mile stretch at Chiho Hsien, 15 miles up stream from Lokou, where there is scarcely more than 1 500 ft between the protected dikes); but this stretch is too narrow and should be widened. The observed water levels indicate that this section causes a considerable heading up during heavy floods such as occurred in 1937.

When it comes to dike distance width and channel width for the case in which a detention basin system has been built up stream, quite different considerations must be met. In this case the outflow from the detention basin at its maximum, for one or two days, will have a relatively light silt load. As the water level becomes lowered in the basins, the deposited silt begins to be scoured, causing the outflow to be heavily silt-laden. One may be reasonably certain that there will be heavy scour along the regulated channel during the period of less silt-laden outflow, and, therefore, it will be feasible to have a much narrower channel than that required under an assumption of no

scour of the channel. A channel width in the up-stream reaches not exceeding 1 800 ft seems amply sufficient; but the same foreshore widths should be kept as in the case of unregulated flood flow. When the silty flow arrives the discharge has already become so low that it will be contained in the channel without flooding the foreshores. Gradually, however, the scoured channel will be refilled and this may again raise the water level and flood the foreshores. Silt will then be transferred to them, and this again will retard further deposition in the channel bed. The swifter velocities in the narrow channel will also help to transport the silt load without undue deposition. Having several detention basins, there will also be silt deposition below each dam as well as along the temporary silt storage area between Mengtsin and the Peiping-Hankow Railway Bridge. This will reduce the possibility of any extremely high silt concentration entering the diked regulated channel as a result of the silt depositions in the basins being scoured out. The rise of the river bed which has been observed in the upper reaches of the diked channel during small, medium silt-laden freshets, is unquestionably due to the present bed, which is far too wide.

The considerations that have been dwelt upon in the foregoing discussion are some of those which unavoidably arise when an investigator attempts to unravel the perplexities encountered in solving the regulation problem of the Yellow River. It will be noted that no attempt has been made to correlate velocities with silt transportation and bed scour. Velocity is only one of the many factors that enter into this problem and cannot be singled out as the safe guiding criterion. Recent studies in mechanics of turbulence seem much more likely to clarify the situation than mere studies of velocities. The writers have tried to use Reynolds numbers to express efficiencies in silt transportation; but the results are meaningless, chiefly on account of the rapidly changing hydraulic radius. For rivers, a different criterion than the Reynolds number must be found to correlate turbulence and its effect on silt transportation and bed scour.

The question of using dredges as an aid in improving the capacity in the river to carry its silt load has been considered. Since the silt which from year to year is left in the channel is small compared with the total silt transportation it may seem worth while to experiment with specially designed, powerful dredges, to see what can be done, before each freshet season, to remove deposits in the channel which would seem to interfere with channel efficiency and hence silt transportation. Dredged cuts, rapidly done, through such deposits may help in their removal by the flood flow when it arrives. Such cuts may be entirely obliterated during the freshet season, but, nevertheless, may have performed their duty in slowing up the rise of the bed. It will be a new departure in the aims of dredging operations and may incidentally assist navigation.

Channel Stabilization Methods.—Until the main dikes have been moved to their designed position in the new regulation plan it will be necessary (since the Yellow River changes its course so rapidly) to construct some cheap, temporary stabilization works to keep the present channel in place until the new dikes have been built and the permanent channel works begun. The

actual regulation work should begin near the Grand Canal crossing, each year continuing systematically up stream according to a well-formed plan. Below the Grand Canal crossing the river is in fairly good condition over long stretches, and the main improvements will be to ease a number of sharp bends and bring the dike system to a more uniform width. The existing protection should be utilized to the greatest extent and, as much as possible, be made to fit into the general stabilization plan.

When it comes to stabilizing the much wider channel up stream of the Grand Canal crossing the question naturally arises as to which of the following methods will be adopted: (1) A continuous revetment of the channel banks in the concave sides of the bends and where it is necessary to guide the flow in the cross-overs from one bend to the next; (2) a system of groins; or, (3) a combination of Methods (1) and (2).

Method (3) will probably be the most logical since there naturally will be many places where the banks are fairly stable and will merely require paving and under-water rip-rap for complete stabilization. In such places it will be necessary to construct long groins tied to the dikes. Furthermore, in bends where the flow necessarily comes close to the dike, and where no previous groin work exists, it will be preferable to use the continuous revetment method. Groins in bends create a highly pulsating flow condition during floods and many groins have collapsed from this cause. A certain degree of turbulence is necessary in order to carry the high silt content in suspension, but this should be in moderation. Excessive resistance merely creates higher water stages. The bends themselves unquestionably produce sufficient eddy currents to propel the silt in suspension past the bends and, therefore, a smooth flow around the bends should be sought.

A question of a more difficult nature arises where the channel must be stabilized through reaches where the soil is easily scoured and unstable and the channel far removed from the dikes. Long low groins will be necessary in such cases to protect the foreshore lands and the stabilization works must be tied to the dikes. Several forms of such groins suggest themselves, such as the pile groin, the fascine-stone groin, and the protected earth groin. The first two forms are indispensable where the existing channel must be narrowed, and the earth groin may be used to tie across foreshores, which are dry at medium stages. However, as an earth groin must be above high water to escape destruction, its use will probably be very limited. The guiding principle, in all cases, should be such that silt flow on the foreshore land will be fairly unobstructed in order to give facilities for the channel to scour; but the velocities on the foreshore land must not be so swift that they will tend to scour side channels.

Availability and cost of materials will determine whether the fascine-stone groin should be used to bridge the space between the banks and the dikes, or the pile-hurdle groin. The latter will require a mattress construction along its full length to prevent its being undermined and destroyed. The outer ends of these groins should define the new channel alignment, and should be in the form of stone heads. Since economical considerations make it inad-visable to space these long groins close enough to keep the channel from

becoming sinuous, it will be necessary to construct short, intermediate groins to prevent this effect. These intermediate groins may be exactly long enough to be tied to the nearest normal bank or ground, which is above the ordinary summer stages, between freshet periods. As regards the question of spacing the groins, and whether they should be short or long, much depends on the soil conditions. Obviously, a sandy, easily scoured soil requires closer groin spacing than if the soil is of a more erosion-resisting quality.

Experience on the Yellow River indicates that nothing but solid construction work will endure the onslaughts of a flood. Miserable failures have resulted from attempts at constructing light pile groins for flood control. They have been laid flat or rooted up by the silty, swift water, and the great bulk of weed root and kaoliang stalk which the flow carries during freshet periods and which pile on to the works. Anything that remains will be entirely removed by the ice during the late winter. In much exposed places the fascine-stone groins will have the best chances for surviving. They can also be added to readily as the silt accumulates on the foreshores.

A question has been raised as to the best direction to give long groins—up-stream pointing, down-stream pointing, or a direction perpendicular to the current. In their report on the Yellow River, the engineers[10] employed by the League of Nations have advised groins pointing up stream. On the silty Missouri River groins pointing slightly down stream or perpendicular are preferred. Two questions seem to enter into this problem: (*a*) The rate of silting on the foreshores; and (*b*) the turbulence designed to produce the best scour of the river bed and the best silt transportation.

It appears to the writers that hydraulic laboratory experiments can do much to solve the question of groin direction. It is fairly obvious that, for various conditions, one angle will be better than another. The correct or incorrect direction may even be sufficient to determine whether or not the Yellow River, in its upper reaches (where the condition of the bed is not influenced by the extension of the delta at the coast), will raise, maintain, or lower its bed. Channel boundary conditions are important functions in determining the relation between frictional resistance and turbulence, and, as the aim must be to have the greatest turbulence with the smallest flow friction, the direction and shape of the groins may play a decided rôle in the bed scour, refill, and silt transportation questions. Although the river is wide compared to the short groins, and the general bed conditions are seemingly more influential in creating turbulence than the groins, it may be quite possible that the groins can influence the bed conditions as well, and hence, in a double sense, the turbulence. It has also been contended by writers on fluid mechanics[11] that the boundary conditions during increasing and decreasing stages of flow are extremely important, and that certain frictional boundary conditions may even cause still water or eddy flow in the boundary layers during a rapidly decreasing flow. In a silty river this explains partly, if not wholly, the rapid refill which occurs as soon as a flood period has passed its crest. That groin

[10] "Hydraulic and Road Questions in China," Series of League of Nations Publications, 1936.

[11] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 463.

shape and direction can modify the refill somewhat is possible and is certainly worth investigating. Extensive hydraulic laboratory experiments to find the best solution to this problem are likely to repay themselves a hundred-fold.

Influence of the Delta.—It has been stated in the "Introduction" that the steady advance of the delta into the shallow Gulf of Chihli causes a flattening of the slope of the river bed in the down-stream section of the diked course, making this higher than the plain. As far as can be estimated from the existing hydrometric records the silt volume that is being deposited on the undiked delta land, and in the neighboring sea, amounts on the average to about 15 000 000 000 cu ft per yr. Previous estimates of 8 000 000 000 to 10 000 000 000 cu ft per yr are unquestionably too low, whereas the observed silt quantity of 35 000 000 000 cu ft passing the Litsin Hydrometric Station in 1934 seems much greater than the average. In 1935 only 12 000 000 000 cu ft passed the Litsin Station; but if the Tungchuang breaches had not occurred, it would probably have been approximately 25 000 000 000 cu ft, a volume still greater than the average under the present condition of frequent dike breaches.

After the diked course has been regulated and well protected the average yearly silt volume reaching the delta and the sea may be as much as 22 000 000 000 cu ft, which means that the delta will advance faster eastward than it does to-day. It does not seem likely, however, that in the near future the river course in the lower diked section will become much more elevated above the plain than it already is. Table 1 shows that the slope from Lokou to Litsin is 1 : 12 600, from Litsin to Holungchu, 1 : 10 000, and from Holungchu to the sea, 1 : 6 000. Toward the end of the Nineteenth Century the dikes were extended from Litsin 10 miles down stream, but it was only during the present decade (1928–1938) that they were continued the next 5 miles to Holungchu. The low-water surface between Lokou and Litsin is almost uniformly 5 to 6 ft higher than the plain. At a point 10 miles down stream from Litsin it is practically level with the plain, while at Holungchu, where the dikes end, the low-water elevation is about 2 ft lower than the delta plain. This merely shows that the dikes east of Litsin have been built on the advancing steep, frontal delta débris fan. Beyond the ends of the dikes the silt spreads and deposits freely in all directions; and, as the débris fan is extended, the slope inside the diked course will be flattened to correspond with that up stream from Litsin. As long as there is distance enough for a steep, frontal débris-fan slope to exist between the ends of the diked course and the sea coast, however, it does not seem probable that the super-elevation of the diked course from Lokou to Litsin will become much greater than it is at present. Hence, the slope of 1 : 12 600 should not become markedly flatter, at least not for some time to come (see Fig. 30).

Surveys conducted by the British Admiralty about 1870, by the German Land Survey Bureau some time after 1900, and by the Chinese Hydrographic Survey Bureau and the Yellow River Commission in 1936 and 1937, show that the delta has advanced 12 miles, corresponding to a rate of one-sixth of a mile per year since the British survey was made. Since the time of the German survey the advance has been about one-fifth of a mile per year. The delta

sector has a radius of approximately 40 miles, and a sector arc-length of 60 miles. The average sea depth has been approximately 40 ft, the under-water slope of the deposited material, judging from recent surveys, is about 1 : 1 500, and the frontal slope of the delta fan as it is to-day—namely 1 : 6 000. This gives a total deposition volume of approximately 1 150 000 000 000 cu ft.

FIG. 30.—ADVANCE OF THE DELTA FAN AND THE GRADUAL DISAPPEARANCE OF THE STEEP FRONTAL SLOPE INTO THE SEA

Since the annual silt volume passing to the delta and the sea has been estimated at 15 000 000 000 cu ft, it has taken almost 76 yr (since about 1862) to deposit the foregoing volume. This is in fair agreement with the advance rate of the delta as given by the surveys, if allowance is made for silt carried beyond the 60-mile arc-length front. The latest Japanese maps, printed in 1935, show the average depth in the Gulf of Chihli (that part lying west of Longitude 121° E) to be about 22 to 23 m or, say, 70 ft. If there is no geosynclinal action interfering with the elevation of the sea floor, the Gulf of Chihli should become dry land (except the part north of Latitude 39° N) within 2 000 yr, assuming that the Yellow River continues to flow into it and has the same silt-carrying capacity as at present. This is an interesting speculation; but what is of more importance is to decide if there is any immediate cause for alarm due to the rapid delta advance. The length of the steep frontal slope of the delta fan from Holungchu to the coast line is 37 miles; and, assuming a rate of advance of the delta under a regulated condition of the Yellow River to be one-fourth of a mile per year, it will take 148 yr for the delta front to have passed completely into the sea. (Actual computation on the basis of 22 000 000 000 cu ft per yr gives 138 yr.) After all, this is not a very long period, and when it has passed, a highly critical situation will rapidly develop which is likely to spell the doom of the present course from the Grand Canal to the sea. Hence, it is important that the advance of the delta should be retarded as far as this is possible. One way would be to try to distribute the silt over a longer front than at present.

The length of the coast line along which the river has spread its silt during the last 70 yr has been stated to be 60 miles. It may be possible to lengthen

this coast line gradually to about 100 miles by having a three-forked diversion point 10 miles below Litsin, one arm going in a northerly direction, one to the east, and the third toward the southeast. Each branch would need to be diked from the diversion point for a distance of 5 or 6 miles, the diking being continued from time to time as the delta fan advances. To prevent any of the branches from developing their sections so that the flow shall become unevenly distributed it would be necessary to construct submerged sills with bank revetment immediately below the offtake point of each branch. (An hydraulic laboratory experiment would be necessary to determine the best divergence angles of the branches to make sure also of an even silt distribution.) Dredges may be used to clear away silt from any of the branches that might tend, temporarily, to silt up. They might also be used to coax the flow with its silt to pass to the sea in as few channels as feasible in order to avoid a flooded condition in the open delta just below the ends of the diked branches. Such deposition frequently takes place to-day and causes silt settling with a rise of the river bed and increased flood stages for a considerable distance up stream from the head of the delta. When the course is good through the delta and most of the silt passes out to sea the diked course up stream again becomes deeper. If the silt can reach the sea it will be distributed better by the littoral currents, which are quite pronounced. It seems natural that the up-stream course should tend to deepen when the delta course is good since, then, there will be a tendency for the flow to scour a deeper channel in the transition zone between the steep and the flat slopes, and by back-cutting, this propels itself up stream. There are indications that this cycle of bed rise and scour, due to good and bad delta channels, influences the diked course as far up stream as the Grand Canal crossing. It is of special importance to have the delta channel in good condition before the flood season arrives as an early serious flood may cause overtopping of the dikes in the lower reaches if it should find the bed in a deteriorated condition and the scour not rapid enough to accommodate the flow. This nearly happened in 1936.

If as much of the silt as possible can be made to reach the sea, two objects are attained: (1) To prolong the life of the steep frontal delta fan slope, which prevents flatter slopes from developing up stream; and (2) to keep the up-stream channel in a deeper condition.

It does not seem technically feasible to construct training works at the mouth to guide the flow and the silt into deeper water where the littoral current could distribute it. Even with this agent acting the silt deposition would be too rapid locally. The training works would also have to be tied to a dike system extended to the coast, which, again, would mean a deposition of the entire silt volume over a limited area. This should absolutely be avoided since it would soon re-act unfavorably on the course up stream. Spreading the silt over a wide front so as to maintain the present course as long as possible seems the best plan. The one advocated is one of several that may be adopted. The Tu Hai Ho by-pass channel which has been proposed should be guided to the north of any branches that may be conducted across the delta.

If nothing is done in the loess areas of the Yellow River water-shed to diminish the soil erosion, there is certain to be serious difficulties in maintaining

the Yellow River on its present course from the Grand Canal to the sea after another two centuries have passed. During the next few decades every effort should be made, therefore, to enforce an energetic soil-erosion control campaign. This promises so much for the Yellow River that no statements recommending it can be too strong.

Control of Soil Erosion.—Were it not for the heavy load of silt in the Yellow River at high stages, the flood-control problem would be far less difficult. The vast quantities of fine loess soil that are washed off the hillsides in Kansu, Shensi, and Shansi, in the rainy season, and are brought into the Yellow River by such feeders as the Wei Ho, the Lo Ho, and the Fen Ho, create a most formidable problem. Therefore, erosion control should be a most important factor in flood control for this river; and, yet, there is ample evidence that it has not been practiced in a sufficiently effective manner. It is true that many farmers have sought to protect their side-hill lands by terracing them and have done so to a degree. However, the thousands of gullies and ravines that have been cut into these loess hills by the agency of running water bear testimony to the wearing away of these lands at an alarming rate.

In his soil studies in China Mr. James Thorp has called attention to this problem,[12] and states that the methods of control adopted by Chinese farmers have been only partly effective, although he finds the terracing in Northwest China to have been conducted over a wide area. Lack of rock supply and timber reasonably close to the regions being treated has made this work less effective in the great loess-covered water-shed of Kansu and Shensi, for example, than in the hilly parts of Shantung where stone walls are used in making lasting terraces. More rigid application of rules as to level terraces, contour plowing, and pit and strip planting, as well as suitable supervision of a drainage system, will all aid in preventing further inroads of eroding processes on the lands that are now used for cultivation. However, in the great problem of controlling the gullies and ravines that are already deeply cut into the landscape, timber and stone must be used in broad areas where these are very difficult to obtain. Here, the power of the State must enter and a large subsidy must be created to aid local farming communities to put these gullies and ravines under control if this erosion is to be seriously and effectively retarded. The native population does not possess the wealth or the leadership to carry through this phase of the program, which will be costly and will require at least two or three decades with Government support of a most substantial character. If the outside support is only moderate, a century or more may pass before this important part of the Yellow River flood control is well in hand.[13]

The field laboratory, established in Western Honan early in 1937 to make certain experiments in connection with gully-erosion control, has yielded few results. Those in charge made certain brief field studies in Japan and, later, were driven from their work by war conditions. The only laboratory work of significance to date in this connection is the great natural open labora-

[12] "Soil Erosion in China," by James Thorp, *Journal*, Assoc. of Chinese and American Engrs., July-August, 1936.

[13] "Soil Erosion and River Regulation with Special Reference to the Yellow River," by S. Eliassen, *Journal*, Assoc. of Chinese and American Engrs., January-February, 1936.

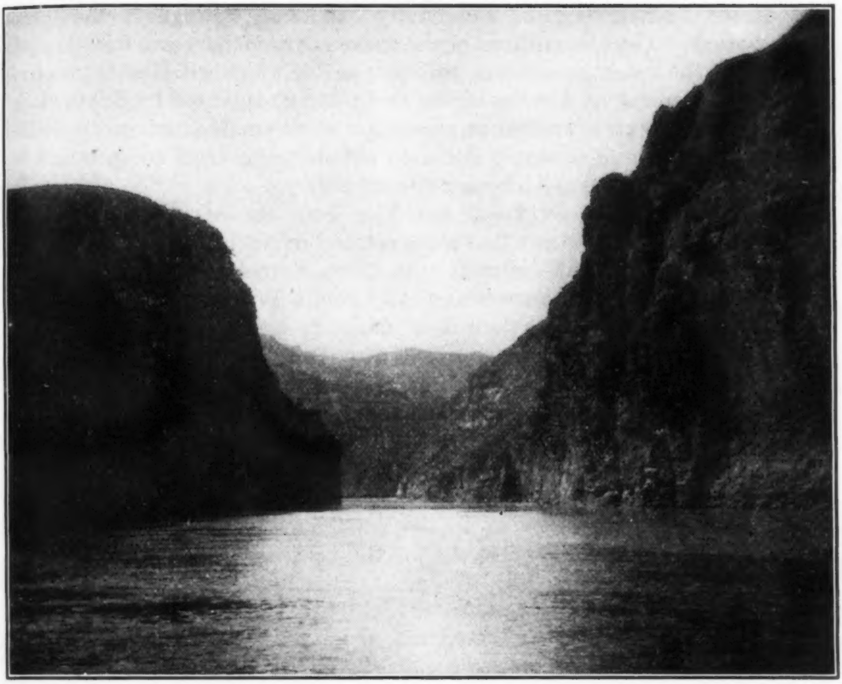


Fig. 31.—Yellow River Gorges 3 Miles Above Yumenkou

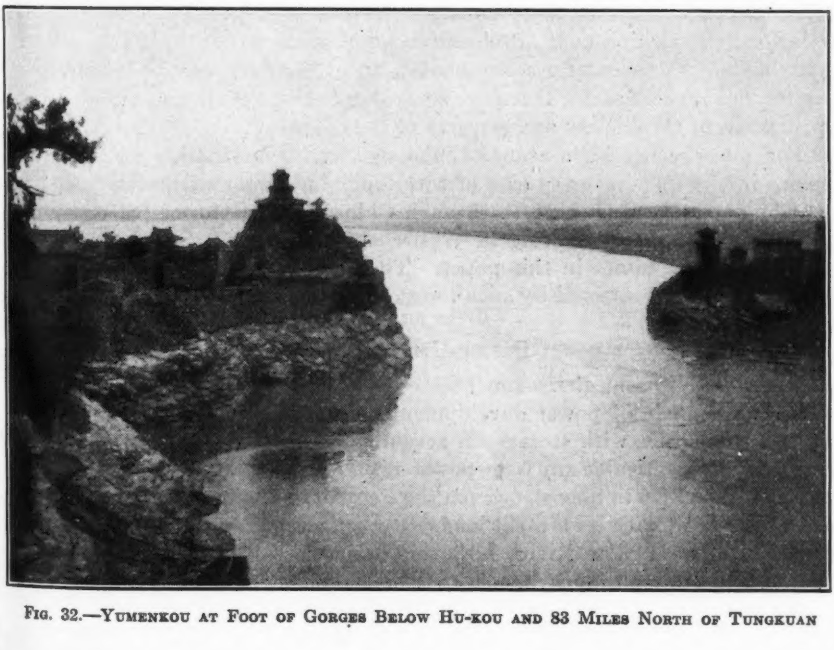


Fig. 32.—Yumenkou at Foot of Gorges Below Hu-kou and 83 Miles North of Tungkuan

tory of the Chinese farming community extending throughout the loess-covered areas. The observations of the writers over many years lead them to believe that the condition of these gullies, many of which will lend themselves readily to treatment by a series of low earth dams supported by cribwork or piling, can be improved within one generation at reasonable cost under intelligent leadership where a way is found to obtain the general co-operation of the various land-owners and strong State support.

Reforestation and Run-Off.—It has long been the hope of students of North China's flood problems that some method might be found whereby the rainfall that is concentrated almost entirely in the months of July, August, and September, with occasional heavy rains in late June, might be spread out over a longer period, or that the run-off might be better distributed through the year. Since storage of flood-waters in reservoirs for weeks or months is not practicable for the Yellow River system, due to the heavy silt content in the water during the rainy season, the remedy of reforestation has been frequently recommended as a panacea.

The speculation that has existed in China as to the effect of extended reforestation on run-off in the Northwest has not produced a unanimous opinion among climatologists as to the possibility of prolonging the rainy season. The group that has insisted for the past two decades that China's flood problems will be remedied, to a great extent, by heavy reforestation in the arid regions of the Northwest, has gradually become less vocal as the difficulty of the problem has been shown, and as more has been published on the matter of storm occurrences due to cyclonic action. To the writers it appears that, although the duration of the rainy season may not be appreciably increased and the climate modified by forests in a way to produce more frequent rains, there may be a distinct benefit in the line of arrested run-off if North and Northwest China ever developed a suitable forest cover. Such a cover over one-half the area of Kansu, Shansi, and Shensi, cannot be obtained at any reasonable cost and will require several decades of persistent effort due to the dryness of the climate and poverty of the country.

For many centuries seasonal typhoons have been coming in from the regions in the Pacific Ocean east of the Philippines and drifting westward to the China coast and then north through China. Other storms belong to the continental system originating in West-Central Asia and drifting eastward, as discussed heretofore in this paper. These typhoons are all of such force as to be only little affected by such forest cover as may develop in the Northwest Provinces.

River Utilization

Power Development at Hu-kou Falls.—Only since about 1933 have serious studies been made of power development on the main course of the Yellow River. Difficulties with storage on account of the rapid silting of reservoirs have made impracticable any large power developments on the main tributaries, such as the Wei Ho in Shensi, or even the Fen Ho in Shansi, although in either case winter flow, with its low silt load, may be stored a few months for spring irrigation. The Yellow River, however, has a sustained low-water flow at Hu-kou Falls warranting hydro-electric development at that point, whereas

along the gorges of the Yangtze, no great industrial city is near-by to use the power thus developed; but the possibility of utilizing power for pumping in connection with irrigation in Southwest Shansi, as well as for industrial uses in surrounding *hsien* cities (a "*hsien* city" is a "county-seat"), seems to warrant the development of the power afforded by this fall in the main river.

The Hu-kou Falls are 125 miles north of Tungkuan, Shensi, or nearly 45 miles north of Yumenkou. After having passed a series of rapids and narrow gorges between Hu-kou and Yumenkou (see Figs. 31 and 32) the river debouches on a plain which lies chiefly in Eastern Shensi, but also includes the valley at the mouth of the Fen Ho, in Shansi. The region to the east of the river, from the Fen Ho south, is a loess plateau of which more than 150 000 acres lie on an average of 250 ft above the river at Yumenkou. This land might be irrigated by pumping from the Yellow River, using low-priced power from Hu-kou Falls. The climate and soil are both well adapted for cotton growing.

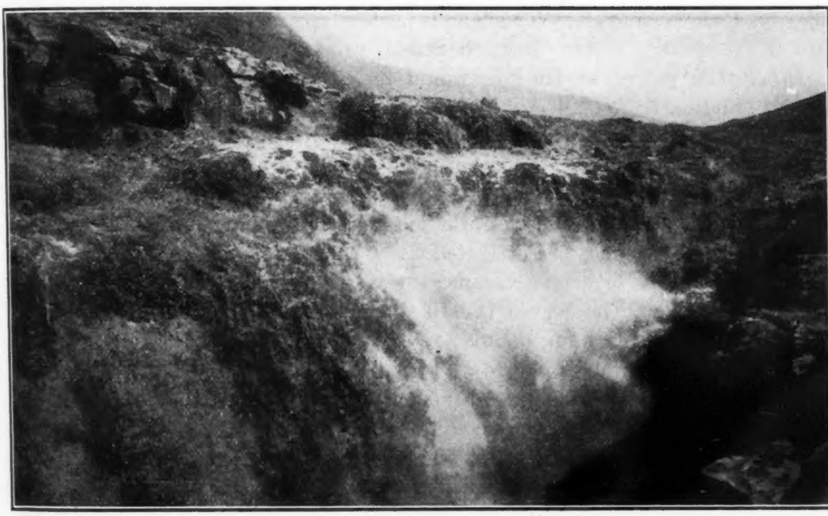


FIG. 33.—HU-KOU FALLS ON THE YELLOW RIVER

In 1934, surveys, studies, and geological investigations were made of the Hu-kou site under the direction of one of the writers. Estimates based on these and other investigations indicate that a development of 100 000 hp dependable every day of the year can be made very economically, costing approximately \$3 000 000 in United States currency, including transmission to Yumenkou and vicinity. With a diversion dam 40 ft above the river bed and 600 ft up stream from the Falls (Fig. 33) the available head at the power site, chosen 5 000 ft down stream, would be a little more than 110 ft. If a flood-control detention basin is to be established above these Falls, the problem of combining such flood-control project with the hydro-electric power development remains to be considered. Probably it can be solved by letting

the lower 40 ft of the dam act permanently as head for power while the higher dam section will serve for flood-control purposes.

Irrigation.—A thorough program of control for the Yellow River will include improvements to existing gravity irrigation projects, such as those near Ninghsia, the Ho Tao region, and Saratsi. In addition to them, however, two projects already mentioned might be undertaken that would require pumping on to table-lands in Southwest Shansi, immediately south and east of Yumenkou. One of these projects, near Hotsin, was surveyed in 1934 under the direction of one of the writers, and its construction has been strongly urged by the local officials. In embraces 50 000 acres of cotton land on terraces and sloping plains from 200 to 300 ft above the Yellow River at Yumenkou. A larger tract lies across the Fen Ho to the south of Hotsin on a plateau, where approximately 200 000 acres, lying at an average elevation of 250 ft above the low-water stage of the Yellow River (where the Fen Ho meets it), offers irrigation possibilities. As yet (1938) no reliable surveys of this area have been made. This land is particularly adapted to cotton growing, but it frequently suffers from deficient rainfall. Development of cheap hydro-electric power at Hu-kou would make the building of an irrigation system practicable.

Ten miles above the Peiping-Hankow Railway the Yellow River might be tapped on the north bank for a gravity irrigation project that would parallel the river to the east. Although surveys have not been made for laying out canals for such a project, studies of available maps indicate that an area of at least 200 000 acres may be irrigated at a reasonable cost. This irrigation would insure crops of cotton, beans, wheat, etc., in drought years. Most of the land that would be served thus is of good quality for mixed agriculture and can maintain a farming population of 1 000 per sq mile.

An indispensable accessory to all irrigation plans for the Yellow River in the plain is that the river channel must first be stabilized where it runs past the intake works. If not, the river is certain, after a few years, to form a high foreshore in front of the intake due to silt deposition and change of course.

Fertilization by Flooding.—The dike break of 1935 in Western Shantung carried large quantities of fine sand out of the river bed across country to the southeast. Inspection trips over this flooded region were made by one of the writers both in June and in October, 1936, to estimate the damages and the benefits due to sand and good silt deposits. The heaviest sand deposit followed the course of the principal channel eroded by the flood waters for a distance of nearly 50 miles making almost worthless for agriculture approximately 350 sq miles of farming country that, in previous floods, had sustained similar injury, but had again been improved by fertilization and farming. The October inspection trip showed that the late spring planting on these lands yielded scarcely enough to pay for the seed. Farther away from the break where there had been ponding with but little current, deposits of very fine silt or loess had greatly improved the land. Over large areas 50 to 60 miles from the break such deposits of clay-like silt were from 1 to 2 ft in depth, but not, as a rule, dry enough for planting in 1936.

It seems probable that systematic flooding of these low, sandy lands by the use of adequate gate structures, that will retain most of the sand within the river bed while accumulating a deposit of fertile silt over the area, will prove feasible and beneficial. A zoning plan can be developed so that each year a district on each side of the river may benefit by such controlled flooding. Five or six years later those same areas would have another turn to receive fertile mud from the river through controlled flooding at times when the suspended silt load is high.

Attempts have already been made in Eastern Shantung to siphon Yellow River water on to alkali lands near the sea; but even with the 24-in. cast-iron pipes thus used the process is slow. Flooding through an extensive system of gates seems necessary if this land improvement is to be undertaken seriously.

Navigation Improvements.—At present, only light-draft junks can ply the Yellow River in the low-water season. A power launch drawing more than 2 ft would have difficulty navigating the diked section of the river in low-water stages. With the proposed training of the channel, perhaps, depths slightly exceeding 3 ft may be maintained at all stages, although bars will continue to be built in new places as the bed-load moves and new silt deposits are laid down. This applies to the river from the Peiping-Hankow Railway to the sea. In order to have depths greater than 3 ft it would be necessary to construct a system of contraction works inside the proposed regulated channel. This would interfere with the river's scour action, however, and would prevent the enlargement of the channel so necessary for carrying the flood flow safely. Perhaps it would also encourage increased refill which would cause the bed to rise more quickly than otherwise. The only navigation improvement worth while would seem to be a light system of construction works at places which show tendencies to much shoaling. These works would be washed out by the first freshets and would have to be replaced early each spring. Of course, the deepest channel would have to be marked constantly.

From the Peiping-Hankow Railway to Tungkuan, the country on both sides is mostly very mountainous and sparsely populated. The currents are swift on account of the river's steep slope, making up-stream navigation slow and difficult. The treacherous Sanmen Rapids are also much feared by the boatmen. Before the construction of the Lunghai Railway this part of the river carried a large part of the exports from Southeast Kansu, West Shansi, and Central Shensi; but the railway has now practically absorbed all the traffic and there is left only a small tonnage of coal transportation from mines along the banks below the Sanmen Rapids. Navigation through this section is now unimportant and not worth considering.

From Tungkuan to Yumenkou no very swift-water stretches are encountered, but immediately to the north of Yumenkou there are gorges where the river is very narrow, and the rock walls are nearly vertical in many places (see Fig. 31). Above Yumenkou there are a number of important coal mines from which the Lunghai Railway and large areas in Central Shensi draw their coal supplies. At high-flood stages no craft can go up these gorges and, even in low water, the coal barges have difficulty in getting through the 15-mile

stretch from Yumenkou to the coal mines above the gorges. Good power boats may be feasible here as the traffic seems to warrant their use.

For the next 30 miles to Hu-kou Falls the river has a number of rapids. Boats rarely go up this stretch, therefore, although they may shoot these rapids on their way down stream after being hauled around the Falls. In consequence, only cheaply constructed shells are made for traffic through this section and such boats are sold in Honan or Shantung as it is not economical to make the return trip

The question of getting boats around the Hu-kou Falls has been a problem for years The passage is highly dangerous except for a short season of the year when unloaded cotton boats may be taken down a small channel prepared for that purpose. All cargo is carried past the Falls by donkeys, a distance of two miles, and then reloaded. As yet the construction of modern locks to handle boat traffic past these Falls has not been seriously discussed by the authorities.

Other rapids 200 miles farther north make navigation of this part of the river dangerous in low water. Sharp rocks often tear holes in boats that attempt to pass down this section. Pilots must use the greatest caution and pick a medium stage of river for travel.

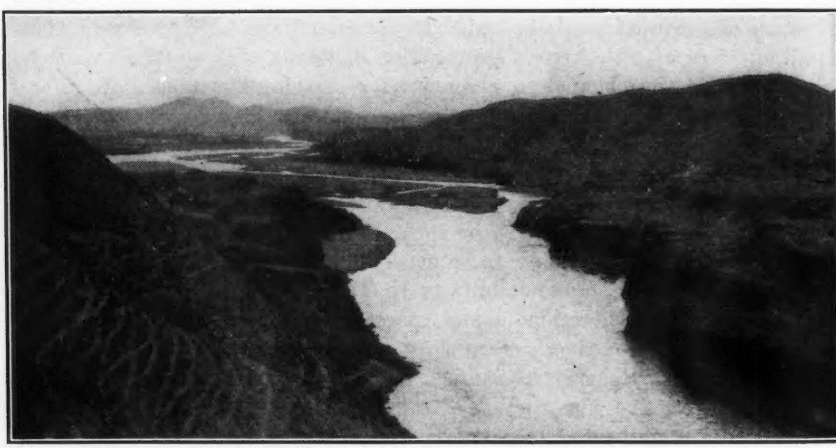


FIG. 34.—THE YELLOW RIVER 20 MILES BELOW LANCHOW, KANSU, 1931

Although Shansi officials have discussed improvements to navigation, so that grain boats may go from Ninghsia and Suiyuan to points in their own Province, the regulation of the Yellow River along the west border of Shansi has appeared too formidable and has not been undertaken. The problem is worthy of further study, since water transport may prove cheaper than rail haulage if it can be handled safely and without costly portages. It would not be an all-year service due to the long ice period and the suddenly appearing freshets of the high-water season, but it might be relied upon for six months of the year.

An interesting form of navigation exists between Lanchow and Paotou. In the spring, the natives at Lanchow construct rafts of ox hides stuffed tightly with wool or sheep skins, inflated with air, and lashed to a top framework of light wooden poles. As many as 120 ox hides, or 500 sheep skins, are used for each raft. It is down-stream navigation only, but as much as 20 tons of goods have been carried on a single raft. The passage through the narrow, jagged gorges between Lanchow and the Ninghsia border is rather perilous (see Fig. 34) and many rafts have been wrecked in this stretch; but the overland journey to the coast is so long and expensive that the dangers of the river route are risked. On arrival at Paotou the wooden frame is sold and the skins deflated and carried overland back to Lanchow. Two trips per year are all that can be made by one crew.

Boats are also used on the stretch from Paotou to the Kansu-Ninghsia border for down-stream as well as up-stream navigation, but they cannot ascend the gorge section to Lanchow. It is possible, but very costly, to improve the gorges so that boats may reach Lanchow. Under present trade conditions of Northwest China, however, such heavy expenditures are not warranted.

Steam or motor-driven launches which have been tested on the Yellow River have always encountered trouble because the silt wore out all bearings in contact with the water. Therefore, bearings that can be flushed by clean water under pressure must be adopted. Stern-wheelers driven by Diesel engines may prove the most useful form of engine-driven boats, as the stern wheel is less likely to be interfered with by all the long weeds and roots in the water during floods. The axle-bearings are also above water.

A boat with an air propeller has recently been tried with some measure of success between Paotou and Ninghsia; but maintenance trouble has been a drawback. It seems that lack of experience with engines and their upkeep has been the main reason why mechanically propelled crafts have not been a success on the Yellow River thus far.

The use of dredges for improving the navigable channel along the diked course of the river can only be considered in connection with maintaining a better flood and silt transportation channel as mentioned toward the end of the section on "Flood Control and Regulation: Regulation of Diked Channel Through the Plain."

The regulation of the Yellow River for flood control is a far more pressing problem than putting this river into condition for safe navigation. In fact, the former must precede the latter.

Magnitude of a Comprehensive Program

The area lying within the Great Plain where floods have done so much damage during recorded history, and where, at any time in the future, the population may suffer if adequate control and regulation work is not undertaken, is estimated to have an area greater than 35 000 000 acres, or 55 000 sq miles—approximately the area of the entire Shantung Province (see Fig. 35). The Provinces of Hopei, Honan, Kiangsu, and Anhwei, as well as Shantung, all have plain lands subject to damage by these floods. Parts of all these

Provinces might well bear a moderate annual flood protection tax of 20 cents (U. S.) per acre for a period of two decades or more. If this is done the Central Government will be better able to lend aid to the poorer Provinces of Shansi, Shensi, and Kansu in coping with the soil-erosion problem, which will require three decades or more of persistent work to solve in a satisfactory manner.

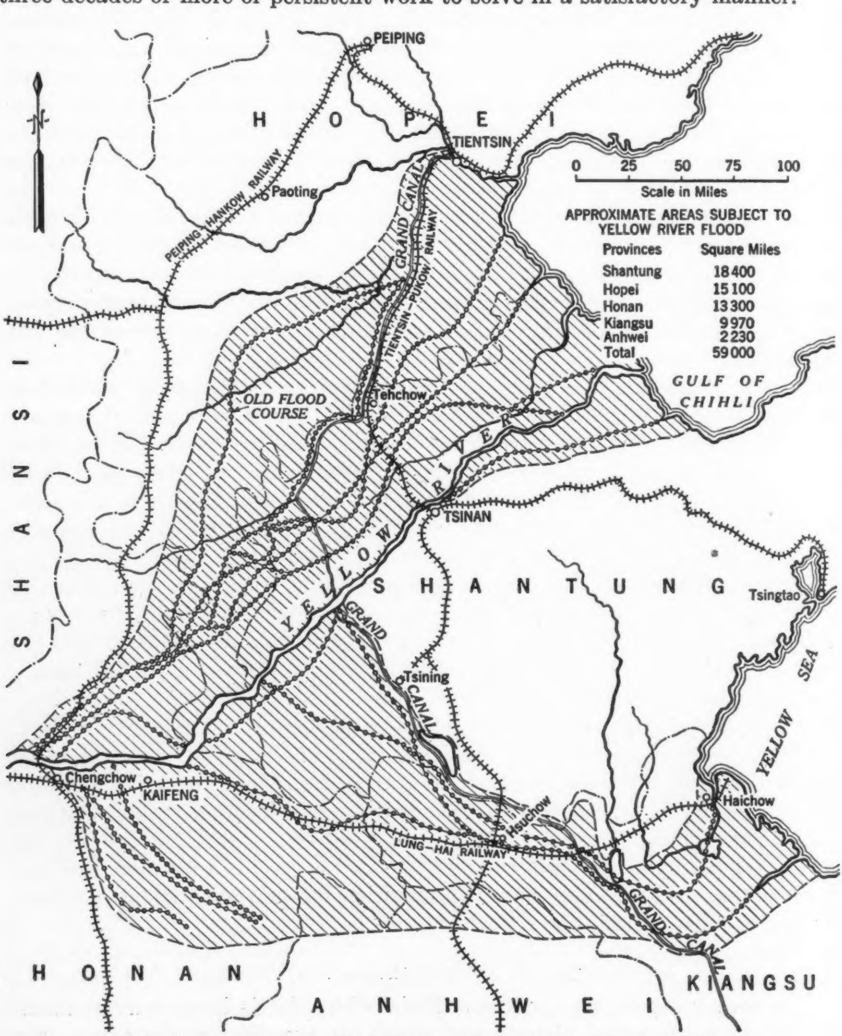


FIG. 35.—APPROXIMATE AREAS THREATENED BY YELLOW RIVER FLOODS

Since construction costs in China are far less than those in America the necessary work can be done very reasonably, and Yellow River floods, such as have cursed North China in the past, can be made so highly improbable in the future as to be no longer dreaded by the people who live within their

reach. The Great Plain of China can be made a perfectly safe place in which to dwell. Furthermore, it can be improved in a number of ways aside from mere flood protection. The follow-up work of soil-erosion control, in the Shensi and Kansu water-sheds particularly, will be possible with continued National Government encouragement; but it is time to begin this problem in earnest. The struggle against these floods has gone on throughout China's history. The means adopted to combat them have only partly solved the problem and then only for very limited periods. Far better insurance is necessary.

ACKNOWLEDGMENTS

The writers desire to acknowledge the assistance of C. N. Joyner, M. Am. Soc. C. E., Deputy Municipal Engineer, British Municipal Council, Tientsin, China, and Dr. Erik Nystrom, Professor of Geology, Shansi University, for their reading of the manuscript and their helpful criticisms and suggestions in connection with the text. Sincere acknowledgment is given also to the Hon. Charles R. Crane, former United States Minister to China. His intense interest in these engineering problems and his continuous support have made possible not only the extensive studies involved, but their publication for the benefit of Americans and Chinese engineers alike.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

EARTHQUAKES AND STRUCTURES

By Leander M. Hoskins,[1] Esq., and John D. Galloway,[2] M. Am. Soc. C. E.

Synopsis

This paper is made up of two sections. Section I, written by Mr. Galloway, refers briefly to the work of the Earthquake Committee of the Society formed in 1923, and to its report. The nature of the earthquake waves is described and some reference is made to their cause, complexity, and characteristics. Some notes are added of the relation between earthquake waves and the structures that may be subjected to the forces arising therefrom.

Section II, by Professor Hoskins, considers the nature of the ground motion, gives certain fundamental rules and conclusions, and investigates the theory of flexure of vertical beams subjected to transverse oscillations. An extension of the theory to an elastic column sustaining a load is given in the Appendix.

I.—HISTORICAL AND DESCRIPTIVE

Historical

In 1923, after the destructive earthquake in Japan, a committee of the Society was formed with American and Japanese members. About five years were spent in assembling data, and a report was submitted to the Society. This report,[3] made up of various chapters, contained, in addition to historical and descriptive matter, a theoretical treatment of the problem of stresses in structures subject to earth movements. One paper, contributed by Professor Joseph N. LeConte, and John E. Younger, of the University of California, at Berkeley, Calif., was published in 1932.[4] The

Note.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **February 15, 1939.**

[1] Prof. of Applied Mechanics, Emeritus, Stanford Univ., Palo Alto, Calif. Prof. Hoskins died on September 8, 1937.

[2] Cons. Engr., San Francisco, Calif.

[3] Available for reference at Engineering Societies Library, 33 West 39th Street, New York, N. Y.

[4] "Stresses in a Vertical Rod When Subjected to a Harmonic Motion of One End", by Joseph N. LeConte and John E. Younger, *Bulletin*, Seismological Soc. of America, Vol. 22, No. 1, March, 1932.

other paper was contributed by Professor Hoskins at the same time and revised in 1926. Section II of this paper by Professor Hoskins, contains a fundamental treatment of the problem of earthquake forces and the resulting effects upon structures subject to earthquake waves. It is a recent abridgment of the paper prepared in 1926.

The Problem

Section I describes the complex nature of the ground movements and the resulting elastic waves. The very complexity of the waves and the form and nature of various structures, such as buildings, water towers, bridge piers and trusses, etc., would seem to preclude any theoretical treatment. It would appear to be beyond the ability of any mathematician to develop formulas that would take accurate account of the extremely variable nature of the factors involved. This is true. On the other hand, the very complexity of the subject makes it necessary to approach the problem by making some general assumptions and to attempt simple solutions; for instance, it is necessary to assume a simple harmonic wave in place of the complex variable waves of the earth when in motion. It is advisable to assume a structure of uniform dimensions and material. Using these assumptions, it is possible to develop equations for moments and shear in accordance with dynamic theory. It then becomes a matter of judgment to establish a relation between any actual structure and the results of the theoretical deductions from simple assumptions that lie within the range of mathematical treatment.

It must not be inferred that more extended theoretical treatment of the problem is impossible. On the contrary, this has been done by others and with good results, especially in buildings. However, it seems desirable to present some of the fundamental conditions and treatment as a foundation for further work.

Earthquakes and Their Effects

Any one charged with the design of engineering structures in any part of the world should take account of possible earthquake forces and the resulting stresses. Although earthquakes are generally confined to the two great belts of the earth, they may occur anywhere.

The earthquake belt that borders the Pacific Ocean passes from New Zealand, through Japan, The Islands of Alaska, California, Mexico, and the Andes, to Cape Horn. It is represented by a ring of high mountains bordering the continental shelf on the edge of the ocean depths. Japan is very close to the greatest ocean depth. There seems to be a relation between the intensity of the shocks and the nearness to great mountain chains and the ocean depths. Some minor earthquakes are volcanic, but most of them are tectonic, the result of adjustments in the crust of the earth. They are evidences of mountain building. It is thought by some that most earthquakes in California are connected with the uplift of the Sierra Nevada and the Coast mountains. The evidences are found in the numerous fault

lines that traverse sections of the State. The earthquake of 1878 was caused by a slip on the Haywards fault. The earthquake of 1906 was caused by a slip on the San Andreas fault where the surface trace was evident for nearly 300 miles. The Japanese earthquake of 1923 was caused by movements in the sea bottom of Sagami Bay about 70 miles south of Yokohama, where an area of about 270 sq miles sank from 300 to 1 300 ft and an area of more than 90 sq miles was elevated from 600 to 800 ft. The earthquake of March, 1933, at Long Beach and other towns in Southern California was caused by a slip on the Inglewood fault.

The rupture of the earth crust, as on the San Andreas fault (the depth of which is estimated to have been 30 miles), resulted in a maximum vertical movement of about 3 ft and a maximum differential horizontal movement of about 22 ft. It is obvious that no structure placed directly over the fault can withstand such a tearing apart of the ground. However, as a rule, the major damage is caused by the waves that travel from the disturbance.

The earth crust is not a perfect solid; it responds to the shock of rupture on the fault by a series of elastic waves that have all the characteristics of waves in a perfect medium. There are three generally recognized types of waves which move outward from the origin with different velocities, suffer reflection and interference, and probably have harmonics. The three groups have the following characteristics:

(1) Longitudinal waves that follow a curved path through the earth crust and move with a velocity of about 3.5 miles per sec;

(2) Transverse or shear waves that traverse the surface layers of the earth and move with a speed of about 2 miles per sec; and,

(3) Waves that carry the maximum amount of energy, travel through the surface, and move with a velocity of about 3 miles per sec.

In addition to these characteristics there is evidence given in almost every shock, of waves that move slowly over the surface of the ground. These waves are seen usually in alluvial soils.

The waves that damage structures are impressed waves, changing rapidly in period, amplitude, and acceleration. Seismograms rarely show traces of harmonic waves, or those with regular characteristics. This is probably a very fortunate circumstance because it prevents, to a large extent, the resonance of vibrating structures. As a result of the impressed forces generating the wave trains, the periods may vary from 0.1 sec to 2 sec, and the acceleration may vary from practically nothing to more than that of gravitation. The amplitude may vary from a small fraction of an inch to 9 in. and, possibly, to 12 in. The foundation material is the important element of the movement. Solid rock moves the least; deep alluvial soils saturated with water, such as the lands bordering the southern part of San Francisco Bay, the plain of the Los Angeles River, or the region from Tokyo to Yokohama, will have the greatest movements.

The duration of the major wave movements is an important element in the problem of the design of structures. Within limits it may be said that this time varies from a few seconds to several minutes. It will be

appreciated that the duration of the destructive movements has an important bearing upon the capability of structures to withstand the earthquake. Reports indicate a duration of the sensible waves of the Long Beach earth movements as 11 sec. At Charleston, S. C., in 1886, the duration was 1 min and 10 sec; in California, in 1906, the duration was said to have been 3.5 min.

The energy released by the breaking of the earth crust is gradually dissipated during the onward progress of the waves. The destructive effects become less as the wave advances and the period of wave tends to increase to as much as 10 sec. The destructive effects are limited to a relatively small area, but the sensible shock may be felt at distances of 1 000 to 2 000 miles. Major earthquakes shake the entire globe and are detected by seismographs now installed in every civilized country.

The earth movements are very complex. All the phenomena of waves in an elastic solid are present, modified by the varying nature of the materials of the crust of the earth. One very important factor is the extreme complexity of the wave action. A movement such as that of the San Andreas fault in 1906, where the earth was ruptured for a length of 300 miles, does not generate a single wave. On the contrary, waves from innumerable foci radiate in all directions. Any given structure may be subjected to movements in all directions. It is this fact, together with the variable amplitude, period, and acceleration of the principal waves, that makes any exact mathematical treatment of structures impossible.

Relation to Structures.—It will be recognized from the foregoing brief summary of the characteristics of earthquake waves, that any structure subjected to such forces must have elements of design differing in many respects from structures subjected only to the vertical force of gravity.

One important element is the vibration period of the structure. All structures have one or more natural periods of vibration. If the period of the structure approximates closely to the periods of some of the maximum earthquake waves of the ground upon which it rests, even if such are variable, there is danger of resonance in the structure. If long continued, the cumulative effects of resonance may result in the destruction of the structure. In general, low, broad structures have vibration periods differing considerably from the periods of the maximum earthquake waves. In the case of high, narrow structures the vibration period is often close to that of some of the maximum waves and, unless the structure is well designed, serious damage will result if the shock is of long duration. The Earthquake Committee filed with its report [3] the vibration periods of a number of buildings in San Francisco, Calif., and the results would be of interest to those having such subjects under consideration. Other data of a similar kind have been published.

It is obvious that the forces generated in a structure subjected to earth movements from an earthquake are measured by the mass or weight of the building. The forces of the moving ground are unlimited. It follows from this that the use of light materials in a structure is the best and that heavy, inelastic materials should be avoided as far as possible.

The form of the structure is of importance when the design is considered. This remark applies with greatest force to buildings. A central section, with wings at right angles, suffers extensive damage, especially in the central section. The wings vibrate in different directions and with different periods, and batter the central section to its destruction.

Since the forces resulting from an earthquake are, in general, horizontal and dynamic, it follows that the design of a structure must provide for the transfer of stresses from one member to another at the connections. A reversal of stress always takes place. The structure, as a whole, is subjected to torsion and to cross-bending, from which arise tension, compression, and shearing stresses. Provision must be made in the design to resist these stresses.

Section I is presented in an attempt to explain the complex nature of the earth movements and the relation to the form of structures, the design, and the materials used. As stated, an exact treatment of the problem by mathematics is impossible, owing to the complexity of the elements of the problem. On the other hand, it is quite possible to arrive at an approximation to the stresses in a structure such as is given in Section II. If such methods are used in the light of data on possible earthquakes that are now being accumulated in all civilized countries, it will be possible to design a structure that will be consistent and thus will have every chance of passing through an earthquake without excessive damage.

II.—STRESSES AND DYNAMICAL THEORY

Nature of Ground Motion

Rules for the design of structures to resist earth shocks must be based upon definite assumptions regarding the motion of the ground. Probably the assumption most commonly made is that the motion is translational; that is, that all parts of the ground supporting the structure are at every instant moving in the same direction with a common velocity. At every instant all parts of the supporting ground will then have the same acceleration, although its value may be varying more or less rapidly in magnitude and direction.

Although it is probably true that, in many cases, the predominating motion is translational, it is not improbable that cases occur in which there is a rotational component of sufficient amplitude to be a factor in causing dangerous stresses. There is considerable testimony, in fact, to the occurrence of visible surface waves; and although such testimony often alleges wave amplitudes of incredible magnitude, it would seem that a traveling wave crest large enough to be visible must cause an appreciable rotational motion of the ground at any given spot. It will be assumed herein that the ground motion is translational unless otherwise specifically stated. In all cases, however, the solution of the dynamical problem depends upon the same fundamental principle, which it is convenient to state in the form of a general rule of procedure.

Fundamental General Rule

The stress condition throughout the structure at any instant depends upon the accelerations of all particles of the structure at that instant. Assuming these accelerations to be known, the general rule of procedure may be stated as follows:

Assume to act upon every particle of the structure and its contents, a force equal to the product of its mass by its acceleration and opposite in direction to the acceleration. Treat the body as if in equilibrium under the action of these supposititious forces in addition to the actual forces.

The actual forces include: (*a*) Forces, such as dead weight and wind pressure, which are independent of earthquake motion; and (*b*) the supporting forces, with values that may be changed materially by the earth motion. The former are a part of the known or assumed data of the problem, whereas the latter are unknown until computed by the aid of the above rule. Just as in the ordinary problem of structural design, the supporting forces must be determined as a preliminary to the computation of the stresses. The fundamental rule is simply the application to this problem of the general principle of dynamics known as d'Alembert's principle.

Application of General Rule on Assumption of Rigidity

If the structure is assumed to be perfectly rigid, and if it does not leave its supports, every part of it must have, at every instant, the same acceleration as the ground. The application of the general rule, therefore, is simple. Thus, if the ground has an acceleration α in a certain direction, the supposititious force assumed to act upon a particle of mass m would be a force $m\,\alpha$ opposite that direction. In other words, the magnitude of the supposititious force would be the fraction, $\frac{\alpha}{g}$, of the weight of the particle, g being the acceleration due to gravity.

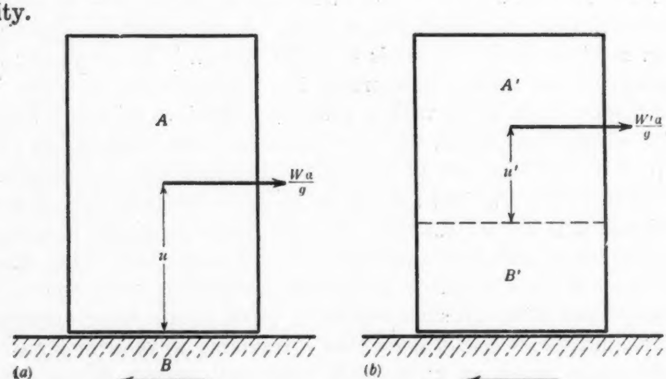

Fig. 1.

To illustrate, suppose A (Fig. 1(*a*)) is a rigid body supported upon the ground B. Let the weight of the body be W lb, and let u be the height of its mass center above the base. At a certain instant let the ground have the ac-

celeration, α, directed horizontally to the left; then the supposititious forces to be assumed to act upon the body would have a resultant of magnitude, $\frac{W\alpha}{g}$ lb, directed horizontally to the right and acting at the mass center. The supporting forces at the base must be such as to counterbalance this resultant; hence, they must be equivalent to a force, $\frac{W\alpha}{g}$ lb, directed horizontally to the left, and a couple of moment, $\frac{u\,W\alpha}{g}$ ft-lb, counter-clockwise. These are in addition to the supporting forces that counterbalance the weight of the body.

In like manner, if A' and B' (Fig. 1(*b*)) are the parts of the body above and below any horizontal plane, the forces exerted upon A' by B' must be such as to counterbalance the supposititious forces acting on the part, A'. If the weight of this part is W' lb, the supposititious forces have a resultant of magnitude, $\frac{W'\alpha}{g}$, directed horizontally to the right and acting at the mass center of A'; hence, the forces exerted by B' upon A' are equivalent to a force, $\frac{W'\alpha}{g}$ lb, acting horizontally to the left (shear) and a couple of moment, $\frac{u'\,W'\alpha}{g}$ ft-lb, counter-clockwise (resisting moment). These are additional to the forces which counterbalance the weight of A'.

If the ground has an acceleration, α, directed vertically upward, the supposititious forces are directed downward; the effect upon the supporting forces and internal stresses is the same as if the weight of every part of the structure and its contents were increased by the fraction, $\frac{\alpha}{g}$, of the actual weight. In like manner a downward acceleration of the ground would have the same effect as a decrease in the weight of the structure and its contents. Whatever the actual direction of the acceleration, its horizontal and vertical components may be treated separately. The following discussion will refer mostly to horizontal accelerations, since it is probable that, ordinarily at least, the horizontal acceleration of the ground is the principal cause of dangerous stresses. (The foregoing statements are true, of course, only when the ground motion is translational. For a rigid body, however, the computation of accelerations throughout the body due to a known rotational motion of the ground can be made without difficulty, thus determining the supposititious forces for applying the general rule.)

Effect of Elastic Yielding

If the acceleration of the ground varied very slowly, the departure of actual materials from perfect rigidity would have no appreciable effect upon the values of the supposititious forces referred to in the general. It would still be practically correct to assume that all parts of the structure have always the same acceleration as the ground. This is no longer true, however, if the ground is in rapid oscillation. It appears in fact that, because of elastic yielding, all parts of the structure above the foundation may have a greater range of motion than the ground, and therefore the accelerations of the upper parts may be

materially greater than that of the ground. The effect of elastic yielding will depend upon the rapidity of the oscillation and upon the elastic properties, dimensions, and mass of the structure. The problem of estimating the yielding is too complex for exact mathematical treatment unless simplified by assumptions which are far from true in the case of many structures. It is possible, however, to infer something regarding the nature of the effects to be expected, from well-known considerations relating to oscillating systems in general. It seems possible, also, to throw some light on the possible order of magnitude of these effects by solving the dynamical problem for idealized simple cases.

Conclusions from General Theory of Forced and Free Oscillations

The problem under consideration falls under the general case of a system which is constrained to oscillate with a certain definite frequency. (By frequency is meant the number of complete oscillations per unit time. Frequency is the reciprocal of period.) General theory indicates that the phase and amplitude of the forced oscillation will depend upon the relation between its frequency and that of the "natural" or "free" oscillation which the system might have. For the simple case of a system having only one degree of freedom and, therefore, one possible mode of vibration, the following statements hold:

(*a*) *Phase of Forced Oscillation.*—If the disturbing oscillation has a less frequency (that is, a longer period) than the free oscillation, the system will oscillate in the same phase as the disturbance that causes it. If the disturbance has a frequency greater than that of the free oscillation, the phase of the actual oscillation will be opposite to that of the disturbance.

(*b*) *Amplitude of Forced Oscillation.*—The amplitude of the forced oscillation is greater the more nearly its frequency approaches that of the free oscillation; that is, if f_0 is the frequency of the free oscillation and f that of the disturbance, the amplitude of the forced oscillation will be greater the nearer $\frac{f}{f_0}$ is to 1. Moreover, as $\frac{f}{f_0}$ approaches 1 the amplitude increases without limit.

These statements refer to undamped oscillations. In the case of an oscillating building the damping (due to internal friction and to air resistance) is presumably too small to affect the validity of the propositions stated. It should further be noted that a system having more than one degree of freedom may have more than one possible mode of free oscillation; for elastic structures the number of possible modes may, in fact, be indefinitely great. In applying Statements (*a*) and (*b*) to such a case it is to be understood that the mode referred to is generally that of longest period, which may be regarded as the fundamental mode.

These statements indicate, at least in a general way, the nature of effects that might be caused in an elastic structure by a simple harmonic oscillation of the supporting ground. Proposition (*b*) is of especial importance, since it indicates that if a structure has a natural oscillation period that is nearly the same as that of the ground motion, continued repetition of the earth tremors will inevitably result in dangerous stresses. Whether this condition is actually

of frequent occurrence, and whether it is possible to guard against it by proper design, would seem to be questions of fundamental importance. To answer the former question with certainty it would be necessary to have full knowledge regarding the frequencies of the harmonic components of ground oscillations occurring in violent earthquakes, and also regarding free-oscillation frequencies of actual buildings. It can scarcely be doubted, however, that observed destructive effects have in some cases been due to the cause stated.

As regards the second question, it is obvious that the designer is powerless to avoid the dangerous condition unless he is able to estimate in advance the natural oscillation frequency of any projected building. Of great value for this purpose would be data concerning actual oscillation periods of many existing structures so selected as to serve as types. The problem of the structural designer in the matter of avoiding dangerous oscillations thus depends very largely upon data which can be obtained only by observation of actual structures and actual earthquakes. It remains to consider whether further light can be thrown upon the problem by dynamical theory.

As already stated, the possibility of exact mathematical treatment depends upon the introduction of assumptions which are often far from representing the facts for actual structures. The solution of such idealized problems, however, may be of use as indicating the general character and order of magnitude of the effects likely to be caused by elastic oscillation. One such problem will now be considered.

Oscillation of a Uniform Vertical Beam

The ideal problem treated in the following analysis is that of a uniform beam placed in a vertical position with the upper end free and the lower end rigidly attached to the ground, the supporting ground being assumed to have a horizontal motion resolvable into components which are simple harmonic functions of the time. The method of solution consists in forming the differential equation of the elastic curve by a simple extension of the ordinary theory of flexure, and solving this equation subject to the conditions assumed to hold at the ends. From the equation of the elastic curve, it is easy to deduce formulas for the shear and resisting moment at any cross-section of the oscillating beam. The solution given herein was formulated by the writer in 1926 when the original draft of this paper was prepared for the Special Committee on Effects of Earthquakes on Engineering Structures (not published). The same fundamental theory has been employed by many writers both before and since that date. A partial list of publications is given at the end of this paper.

Theory of Flexure Applied to Oscillating Beam

The following deduction of the differential equation of the elastic curve is applicable to the general case of a uniform elastic beam performing small transverse oscillations. The notation introduced, however, is especially adapted to the foregoing problem. The quantities involved are the following: h = length of beam; A = its cross-sectional area; $I = A\,k^2$ = moment of

inertia of cross-section about axis of bending; W = total weight of beam, in pounds; E = modulus of elasticity of material (Young's modulus); g = acceleration due to gravity; and, M and V = resisting moment and resisting shear acting at any transverse section.

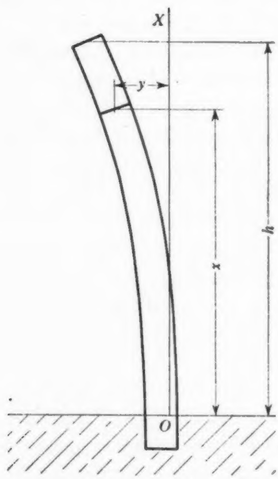

FIG. 2.

Let x, y denote the rectangular co-ordinates of any point in the elastic curve with respect to axes taken as in Fig. 2. The origin, O, is taken at the base of the beam in its mean position, the positive X-direction is vertically upward, and the positive Y-direction is horizontally to the left. The convention of signs for M and V is as follows: The part of the beam above the cross-section exerts upon the part below, forces of which V is the sum of the components in the positive Y-direction and M is the sum of the moments in the counter-clockwise sense.

The first step is to write the dynamical equations for an element of the beam bounded by two transverse sections at x and $x + dx$. The acceleration of this element in the X-direction is inappreciable while its Y-acceleration is $\frac{\partial^2 y}{\partial t^2}$. Its angular acceleration is $\frac{\partial^2 \phi}{\partial t^2}$ in which ϕ differs inappreciably from $\frac{\partial y}{\partial x}$. The mass of the element is $\frac{W\,dx}{g\,h}$ and its moment of inertia about the axis of bending is $\left(\frac{W\,dx}{g\,h}\right) k^2$. The total force acting on the element in the Y-direction is $\left(\frac{\partial V}{\partial x}\right) dx$; the total turning moment acting on it is $\left(\frac{\partial M}{\partial X}\right) dx + V\,dx$. Hence, the equations of linear and angular motion for the element may be written as follows (after cancellation of dx):

$$\frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} = \frac{\partial V}{\partial x} \quad \text{(1)}$$

and

$$\frac{W\,k^2}{g\,h}\frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right) = \frac{\partial M}{\partial x} + V \quad \text{(2)}$$

Eliminating V between Equations (1) and (2), there results,

$$\frac{W\,k^2}{g\,h}\frac{\partial^4 y}{\partial t^2\,\partial x^2} - \frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 M}{\partial x^2} \quad \text{(3)}$$

As in the ordinary theory of flexure,

$$M = E\,I\,\frac{\partial^2 y}{\partial x^2} \quad \text{(4)}$$

so that Equation (3) reduces to the form,

$$\frac{\partial^2 y}{\partial t^2} + \frac{E\,I\,h\,g}{W}\frac{\partial^4 y}{\partial x^4} - k^2 \frac{\partial^4 y}{\partial t^2\,\partial x^2} = 0 \quad \text{(5)}$$

It is found that the third term in Equation (5) is practically negligible unless the beam is short in comparison with its transverse dimensions, and in the following solution this term is omitted. (If the third term is retained the solution is entirely practicable, but involves considerably more labor, especially in numerical applications.) It is to be noted that the omission of this term reduces Equation (2) to the form,

$$V = -\frac{\partial M}{\partial x} \quad \text{(6)}$$

as in the theory of beams under static conditions.

Solution of the Differential Equation When the Ground Motion Is a Simple Harmonic Oscillation

If the ground motion is resolved into components, each of which is a simple harmonic oscillation, each component may be treated separately. It suffices, therefore, to consider the solution of the differential equation,

$$\frac{\partial^2 y}{\partial t^2} + \frac{E\,I\,h\,g}{W}\frac{\partial^4 y}{\partial x^4} = 0 \quad \text{(7)}$$

on the assumption that the lower end of the beam is constrained to have a simple harmonic oscillation in a horizontal plane. This gives one of four boundary conditions; the other three being that $\frac{\partial y}{\partial x}$ is always zero at the lower end, and that M and V are always zero at the upper end. Noting the general values of M and V as given by Equations (4) and (6), the boundary conditions may be stated as follows:

When $x = 0$,

$$y = a \sin\left(\frac{2\,\pi\,t}{T} + q\right) \quad \text{(8a)}$$

when $x = 0$,

$$\frac{\partial y}{\partial x} = 0 \quad \text{(8b)}$$

when $x = h$,

$$\frac{\partial^2 y}{\partial x^2} = 0 \quad \text{(8c)}$$

and when $x = h$,

$$\frac{\partial^3 y}{\partial x^3} = 0 \quad \text{(8d)}$$

The amplitude and period of the forced oscillation are here represented by a and T, and q is a constant the value of which depends upon the choice of the origin of time.

It is easily verified that the differential equation and the four boundary conditions may be satisfied by assuming:

$$y = (A_1 \cos m\,x + B_1 \sin m\,x + A_2 \cosh m\,x + B_2 \sinh m\,x) \times \sin\left(\frac{2\,\pi\,t}{T} + q\right) \quad \text{(9)}$$

and assigning proper values to the constants m, A_1, B_1, A_2 and B_2. Thus, the differential equation is satisfied if,

$$m^4 = \frac{4\,\pi^2\,W}{E\,I\,h\,g\,T^2} \quad \text{(10)}$$

and the boundary conditions give the four equations,

$$A_1 + A_2 = a \quad \text{(11}a\text{)}$$

$$B_1 + B_2 = 0 \quad \text{(11}b\text{)}$$

$$-A_1 \cos m\,h - B_1 \sin m\,h + A_2 \cosh m\,h + B_2 \sinh m\,h = 0 \quad \text{(11}c\text{)}$$

$$A_1 \sin m\,h - B_1 \cos m\,h + A_2 \sinh m\,h + B_2 \cosh m\,h = 0 \quad \text{(11}d\text{)}$$

which determine A_1, B_1, A_2, B_2, when m is known, their values being as follows:

$$A_1 = \frac{a}{2}\left(1 - \frac{\sin m\,h \sinh m\,h}{1 + \cos m\,h \cosh m\,h}\right) \quad \text{(12}a\text{)}$$

$$B_1 = \frac{a}{2}\left(\frac{\cos m\,h \sinh m\,h + \sin m\,h \cosh m\,h}{1 + \cos m\,h \cosh m\,h}\right) \quad \text{(12}b\text{)}$$

$$A_2 = a - A_1 \quad \text{(12}c\text{)}$$

and,

$$B_2 = -B_1 \quad \text{(12}d\text{)}$$

Formulas for M and V are easily written from Equations (4) and (6).

Case of Free Oscillation

If the ground is at rest the foregoing solution is still valid with $a = 0$. In this case Equations (11) determine only the ratios of the four constants A_1, B_1, A_2, and B_2, their actual values containing a factor with an arbitrary value. Moreover, in this case the equations can be satisfied only by restricting T to certain values. For by eliminating A_1, B_1, A_2, and B_2 from Equations (11) there results the equation,

$$1 + \cos m\,h \cosh m\,h = 0 \quad \text{(13)}$$

which must be satisfied by any permissible value of m, while T depends upon m according to Equation (10).

Equation (13) is satisfied by each of an infinite series of values of $m\,h$, which may be determined by trial to any desired degree of approximation. In order of magnitude the first three values are 1.8751, 4.6944, and 7.8548; the

succeeding values differ very slightly from $\frac{7\pi}{2}$, $\frac{9\pi}{2}$, etc. Each of these values corresponds to a possible free oscillation, the period of which may be computed by Equation (10). The greatest of the possible free periods may be called the fundamental period, a formula for which may be obtained by taking $m = m_0 = \frac{1.8751}{h}$. Calling this period, and the corresponding frequency, T_0 and f_0, and using Equation (10),

$$T_0 = \frac{1}{f_0} = \frac{2\pi}{m_0^2}\sqrt{\frac{W}{E\,I\,h\,g}} = \frac{2\pi}{(1.8751)^2}\sqrt{\frac{W\,h^3}{E\,I\,g}} = 1.787\sqrt{\frac{W\,h^3}{E\,I\,g}} \ldots (14)$$

It may easily be verified that the successive free oscillations have periods the ratios of which to T_0 are: 1, 0.1595, 0.05699, 0.02908, 0.01759, etc. These numbers, beginning with the fourth, are to a close approximation proportional inversely to the squares of the odd numbers 7, 9, 11, etc.

Forced Oscillation of Assigned Period

It is seen that, for a given beam, the assignment of values to a and T determines all the constants, m, A_1, B_1, A_2, and B_2, so that (except in the case of free oscillation) the values of y, M, and V, at all sections, may be computed by Equations (9), (4), and (16). In examining the nature of the results for various particular cases it is convenient to take as a basis of comparison the ideal case of no elastic yielding.

Case of Perfect Rigidity.—For this case let the values of y, M, and V, for any value of x, be y', M', and V', and let values for $x = 0$ be y_0', M_0', and V_0'. Denoting the ground acceleration by α and reasoning as before (see heading "Application of General Rule on Assumption of Rigidity") there result the formulas:

$$M_0' = -\frac{W\,h}{2\,g}\alpha \ldots\ldots (15a)$$

$$V_0' = -\frac{W}{g}\alpha \ldots\ldots (15b)$$

$$M' = M_0'\left(1 - \frac{x}{h}\right)^2 \ldots\ldots (15c)$$

$$V' = V_0'\left(1 - \frac{x}{h}\right) \ldots\ldots (15d)$$

Since the motion of the ground is assumed to be represented by the formula,

$$y_0' = a \sin\left(\frac{2\pi t}{T} + q\right) \ldots\ldots (16)$$

the acceleration is,

$$\alpha = \frac{\partial^2 y_0'}{\partial t^2} = -\frac{4\pi^2}{T^2} a \sin\left(\frac{2\pi t}{T} + q\right) \ldots\ldots (17)$$

Effect of Elastic Yielding.—Equations (4), (6), (9), (10), and (15) lead easily to the following formulas:

$$\frac{y}{y_0'} = \frac{A_1}{a}\cos m\,x + \frac{B_1}{a}\sin m\,x + \frac{A_2}{a}\cosh m\,x + \frac{B_2}{a}\sinh m\,x \ldots (18a)$$

$$\frac{M}{M_0'} = \frac{2}{m^2\,h^2}\left(-\frac{A_1}{a}\cos m\,x - \frac{B_1}{a}\sin m\,x + \frac{A_2}{a}\cosh m\,x + \frac{B_2}{a}\sinh m\,x\right) \ldots (18b)$$

$$\frac{V}{V_0'} = \frac{1}{m\,h}\left(-\frac{A_1}{a}\sin m\,x + \frac{B_1}{a}\cos m\,x - \frac{A_2}{a}\sinh m\,x - \frac{B_2}{a}\cosh m\,x\right) \ldots (18c)$$

Equations (18*b*) and (18*c*), compared with Equations (15*c*) and (15*d*), show the effect of elastic yielding upon the values of the bending moment and shear at any section of the beam. It is seen that, for a given value of $\frac{x}{h}$, the values of $\frac{M}{M_0'}$ and $\frac{V}{V_0'}$ depend solely upon the value of $m\,h$. Moreover, $m\,h$ depends only upon the value of $\frac{T_0}{T}$, since by Equation (10),

$$\frac{m^2\,h^2}{m_0^2\,h^2} = \frac{T_0}{T} \ldots (19)$$

while $m_0\,h$ has the fixed value 1.8751, so that,

$$m\,h = 1.8751\sqrt{\frac{T_0}{T}} \ldots (20a)$$

$$m\,x = 1.8751\sqrt{\frac{T_0}{T}}\frac{x}{h} \ldots (20b)$$

Thus, if K is written for $1.8751\sqrt{\frac{T_0}{T}}$, x' for $\frac{x}{h}$, and A_1', B_1', A_2', and B_2' for $\frac{A_1}{a}$, $\frac{B_1}{a}$, $\frac{A_2}{a}$, and $\frac{B_2}{a}$, Equations (18) become:

$$\frac{y}{y_0'} = A_1'\cos K\,x' + B_1'\sin K\,x' + A_2'\cosh K\,x' + B_2'\sinh K\,x' \ldots (21a)$$

$$\frac{M}{M_0'} = \frac{2}{K^2}\left(-A_1'\cos K\,x' - B_1'\sin K\,x' + A_2'\cosh K\,x' + B_2'\sinh K\,x'\right) \ldots (21b)$$

and,

$$\frac{V}{V_0'} = \frac{1}{K}\left(-A_1'\sin K\,x' + B_1'\cos K\,x' - A_2'\sinh K\,x' - B_2'\cosh K\,x'\right) \ldots (21c)$$

in which (from Equations (12)):

$$A_1' = 1 - A_2' = \frac{1}{2}\left(1 - \frac{\sin K\sinh K}{1 + \cos K\cosh K}\right) \ldots (22a)$$

$$B_1' = -B_2' = \frac{1}{2}\left(\frac{\cos K\sinh K + \sin K\cosh K}{1 + \cos K\cosh K}\right) \ldots (22b)$$

All the constants in the formulas thus depend upon the single number K, which itself depends upon $\frac{T_0}{T}$. In other words, for a given value of $\frac{x}{h}$, the ratio of change in M and in V, due to elastic yielding, depends solely upon the ratio of the actual oscillation period to the fundamental free period, being wholly independent of the dimensions and material of the beam, except as these factors determine the value of T_0.

Characteristic Curves.—The graphs of Equations (21) may be called characteristic curves for the problem under consideration. Such curves are shown

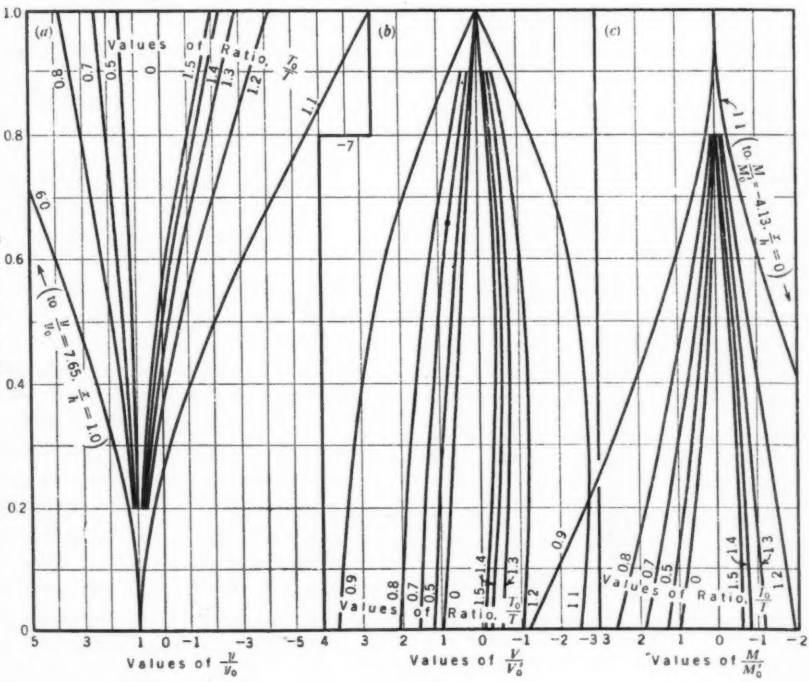

FIG. 3.

in Fig. 3. Each curve is marked with the appropriate value of $\frac{T_0}{T}$, the range of such values being from 0 (perfect rigidity) to 0.9 and from 1.1 to 1.5. The curves illustrate in a striking manner the rôle played by the free period T_0. The foregoing statements (see heading "Conclusions from General Theory of Forced and Free Oscillations"), regarding phase and amplitude of forced oscillations in general are clearly illustrated in Fig. 3(a), and Figs. 3(b) and 3(c) show the rapid increase of M and V as the case $T = T_0$ is approached.

Curves for $\frac{y}{y_0}$, covering a greater range of values of $\frac{T_0}{T}$ are shown in Fig. 4. These curves are of especial interest as showing the transition from the first

to the second of the natural periods. In the case of perfect rigidity, $\frac{T_0}{T} = 0$, so that the ratio of change in M and V, due to elastic yielding, is found by comparing each value computed by Equations (21) with the corresponding value computed from Equations (15). Infinite values of y, M, and V result if T is equal to T_0, or to any one of the possible free periods specified previously. One critical value of $\frac{T_0}{T}$ is 6.238; and it will be noted in Fig. 3 that the values change sign as $\frac{T_0}{T}$ changes from 6 to 7. Coincidence of the forced period with any possible free period is called a condition of resonance.

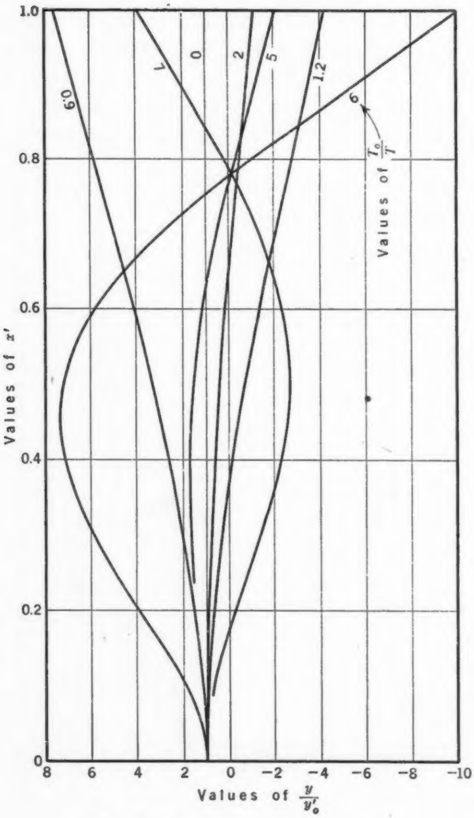

FIG. 4.—TRANSLATORY OSCILLATION.

THEORY OF FLEXURE FOR COMPOSITE BEAM

The theory of flexure, presented herein under the heading "Theory of Flexure Applied to Oscillating Beam," may easily be extended to the case of a beam built of two or more materials, but uniform in the sense that all cross-sections are alike. In such a case, let $E_1, E_2, \cdots$, be the values of the modulus of elasticity for the different materials, and $I_1, I_2, \cdots$, the values of the moment of inertia for the corresponding parts of the cross-section; then, the formula for the resisting moment is:

$$M = (E_1 I_1 + E_2 I_2 + \cdots) \frac{\partial^2 y}{\partial x^2} \quad \text{.............. (23)}$$

The use of Equation (23) instead of Equation (4) introduces no change in the differential Equation (7), except that $E I$ must be replaced by $E_1 I_1 + E_2 I_2 + \cdots$. With this change the case of a composite beam is covered by the solution given above. It is of especial interest to note the application of this statement to the formula for T_0 (Equation (14)). (In the interest of rigor it should be noted that the radius of gyration, k, occurring in Equations (2) and (5) applies not simply to the area of the cross-section, but to the mass of

an elementary layer. In the foregoing solution, however, terms involving k have been treated as negligible.)

Actual and Ideal Structures

It cannot be supposed that numerical results computed for the foregoing ideal problem apply closely to actual buildings, but the general character of the results probably represents the kind of effects to be expected in a steel frame building which is uniform in horizontal area and tall in comparison with its horizontal dimensions. It would seem, for example, that Equation (14) may give some indication of the order of magnitude of T_0 for such a building. The case of buildings that are relatively low and broad might be better illustrated by solutions based upon a somewhat different mathematical theory. For such buildings it is probable that horizontal shearing is more important than bending as a factor in elastic oscillation, and solutions taking account of this factor are entirely practicable. For the purpose of this paper, however, the foregoing solution serves sufficiently as an illustration of the nature of the effects that may result from elastic yielding.

Dynamical Theory as a Guide to Practical Design

Although dynamical theory alone cannot supply definite quantitative rules for the design of earthquake-resistant structures, the theoretical analysis shows clearly the nature of the problem and of the assumptions that must be made as a basis for practical rules. This is perhaps best shown by considering the procedure that would be required in attempting to make a practical application of the theory as it has been presented. In such an application it would be necessary to use specific values of the period and amplitude of the forced oscillation, and also of the fundamental free or "natural" period of the structure under consideration. With these quantities given, the solution of the problem of computing stresses involves the following steps: (*a*) A solution on the assumption that the structure is rigid; and (*b*) an estimate of the effect of elastic oscillation:

Step (*a*).—The solution on the assumption of rigidity is simple and definite, the procedure being to apply the fundamental general rule, assuming every part of the building and its contents to have the same acceleration as the ground. The computation of the supporting forces and of the stresses in the framework is then an ordinary problem in structural mechanics.

Step (*b*).—To determine, quantitatively, the ratio of change of the stresses due to elastic oscillation would require a solution of the elastic problem for the actual structure similar to that which has been given for the ideal structure treated as a uniform beam. In the ideal problem it was found that elastic yielding increased the values of V and M by ratios depending upon the value of $\frac{T_0}{T}$, the effects being shown quantitatively in Figs. 3(*b*) and 3(*c*). Although these quantitative results cannot be assumed to hold for the actual problem, it is reasonable to assume that certain important characteristics of the results do hold; namely (1) that the values of V and M are increased by elastic yielding

and (2) that the ratio of change in V and in M depends upon the ratio $\frac{T_0}{T}$ and increases without limit as T approaches equality with T_0, or with any other of the natural periods of the structure.

Dynamical theory thus makes clear the nature of earthquake hazards, and at the same time shows the difficulty of formulating practical rules for meeting the hazards because of uncertainty regarding the actual data that should be assumed to apply to any particular case. There is one fundamental practical rule, however, which may be stated with confidence:

Fundamental Practical Rule.—The structure should be designed to resist forces, $m\,\alpha$, acting on all its particles in any horizontal direction, α being the probable maximum ground acceleration and m the mass of any particle.

The value of α to be used in applying this rule is a matter for the judgment of the engineer, guided by records of actual earthquakes. Different authorities have recommended values from 2% to 10% of gravity.

Danger of Resonance.—It is clear that very great stresses may result if there is a near approach to a condition of resonance; that is, equality of T to one of the natural periods of the structure. Since any given structure has a multiplicity of natural periods, and since the harmonic analysis of seismograph records has shown a considerable range of values of T, it seems impracticable to formulate any simple rule for minimizing the probability of dangerous stresses due to resonance.

Publications Dealing with the Vibration of Prismatic Bars

The theory of the elastic vibration of prismatic bars has been discussed by many writers during the past 200 yr, among the earliest having been Daniel Bernoulli (about 1743) and his contemporary, Bernhard Euler. The deduction of the differential equation of the elastic curve, in the form given in this paper when gravity is neglected, and its solution subject to stated boundary conditions, are found in numerous publications, of which a few are listed subsequently. Of these, Nos. (5) and (6) treat the same case of the oscillating vertical beam as that treated in this paper, with substantially identical results.

(1) **Strutt, John William (Baron Rayleigh).** Theory of Sound. Second Edition, 1894, MacMillan.

(2) **Love, A. E. H.** A Treatise on the Mathematical Theory of Elasticity. Second Edition, 1906, Cambridge Univ. Press.

(3) **Hort, Dr. Wilhelm.** Technische Schwingungslehre, Second Edition, 1922, Berlin.

(4) **Timoshenko, S.** Vibration Problems in Engineering. 1928, Van Nostrand.

(5) **LeConte, Joseph N., and John E. Younger.** Stresses in a Vertical Elastic Rod When Subjected to a Harmonic Motion of One End. *Bulletin*, Seismological Soc. of America, Vol. 22, March, 1932.

(6) **Creskoff, Jacob J.** Dynamics of Earthquake Resistant Structures. 1934, McGraw-Hill.

(7) **Morse, Philip M.** Vibration and Sound. 1936, McGraw-Hill.

APPENDIX

Extension of Theory Desirable

The theory of the uniform vertical beam or column has served to illustrate the general character of effects which may be caused in structures of a certain type by a forced harmonic oscillation. It seems desirable to extend the theory so as to cover a more general class of cases than has been treated previously. In this more general treatment special attention will be given to the important case of free oscillation.

Elastic Column Sustaining a Load

Let the vertical beam or column treated in Fig. 2 be assumed to carry at its upper end a rigid body weighing P lb. Let the mass center of this body lie in the extension of the axial line of the column at a distance h' from the top; let k' be the radius of gyration of its mass with respect to an axis through its mass center perpendicular to the plane of flexure of the column; and let other data and notation be as in the problem already solved. This case is represented diagrammatically in Fig. 5(a).

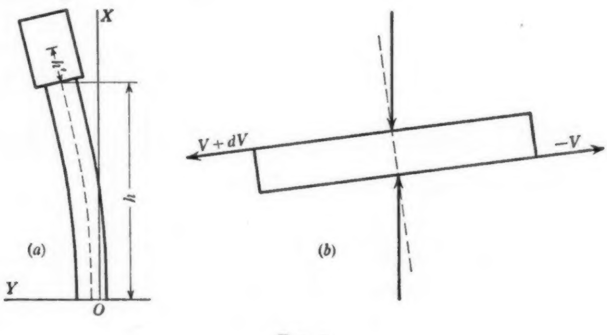

Fig. 5.

Differential Equation of Elastic Curve

In the theory already given the action of gravity on the column was neglected. The following is a more rigorous analysis, taking account of the weight of the column itself as well as that of the supported load. In order to form the differential equation of the elastic curve, write the equations of linear and angular acceleration of an element of the column of thickness dx, bounded by transverse planes normal to the curve. (It will be assumed as before that horizontal shearing is negligible in comparison with bending in determining the form of the elastic curve.) Such an element is represented in Fig. 5(b).

The mass of the element is $\frac{W\,dx}{g\,h}$, and its moment of inertia about its central axis is $\left(\frac{W\,dx}{g\,h}\right)k^2$. Its acceleration in the X-direction is inappreciable, and its

Y-acceleration is $\frac{\partial^2 y}{\partial t^2}$. Its angular acceleration is $\frac{\partial^2 \phi}{\partial t^2}$, in which ϕ differs inappreciably from $\frac{\partial y}{\partial x}$.

The forces acting on the element are as follows: Forces $- V$ and $+ (V + dV)$ acting in practically horizontal directions on the lower and upper faces; couples of moments $- M$ and $+ (M + dM)$, acting on the same faces; a vertical force, $P + W\frac{h - x}{h}$, acting upward on the lower face, and a vertically downward force, less by $\left(\frac{W\,dx}{h}\right)$, acting on the upper face. Each of these vertical forces, being uniformly distributed over the cross-section, may be regarded as applied at the centroid, the distance between their lines of action being dy. (The reasoning here is that of the common theory of flexure, which assumes that the normal stresses on a cross-section are equivalent to a normal force applied at the centroid, together with a couple of which the moment is the resisting moment M.) The equations of linear and angular motion for the element, therefore, may be written as follows (after dividing by dx):

$$\frac{\partial V}{\partial x} = \frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} \qquad \text{(24a)}$$

and,

$$\frac{\partial M}{\partial x} + V + \left(P + W\frac{h - x}{h}\right)\frac{\partial y}{\partial x} = \frac{W\,k^2}{g\,h}\frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right) \qquad \text{(24b)}$$

Eliminating V and substituting for M its value $E\,I\,\frac{\partial^2 y}{\partial x^2}$, the following is obtained as the differential equation of the elastic curve:

$$E\,I\frac{\partial^4 y}{\partial x^4} + \frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} + \left(P + W - \frac{W\,x}{h}\right)\frac{\partial^2 y}{\partial x^2} - \frac{W}{h}\frac{\partial y}{\partial x} - \frac{W\,k^2}{g\,h}\frac{\partial^4 y}{\partial t^2 \partial x^2} = 0 \qquad \text{(25)}$$

Comparing Equation (25) with Equation (5), it is seen that the latter consists of the first, second, and last of the terms in the former, while in Equation (7), which was actually solved, only the first two terms are retained. Next, consider whether a more rigorous treatment which retains some or all of the omitted terms is practicable or desirable. For this purpose, note the meanings of the different terms as well as their relative magnitudes.

Any given element of mass influences the oscillation of the system in two ways—by its inertia and by its weight. The latter effect was wholly disregarded in the theory leading to Equation (5). In the problem there treated (in which the only mass present was that of the column itself) the effect of gravity is represented in Equation (25) by the terms: $\left(W - \frac{W\,x}{h}\right)\frac{\partial^2 y}{\partial x^2} - \frac{W}{h}\frac{\partial y}{\partial x}$. When these terms are retained, and when $P = 0$, Equation (25) can be solved by development in series, and it is not difficult to prove that the effect of gravity is of slight importance in that problem. In the more general case now

under consideration there is the additional gravity effect due to the load P, and it seems probable that there may be cases in which this effect will be of much greater importance than that of the weight of the column itself. In the analysis which follows, therefore, the term containing P is retained but those terms representing the effect of gravity on the column itself are omitted.

The inertia effect is represented in Equation (25) by the second and last terms, of which the former represents the linear inertia of the mass element and the latter its angular inertia. Of these terms the latter is ordinarily of negligible importance and will not be retained. The inertia effect of the load P does not affect the differential Equation (25), but is included in the boundary conditions to be given subsequently.

With the approximations stated the differential equation to be solved reduces to the following:

$$E\,I\,\frac{\partial^4 y}{\partial x^4} + \frac{W}{g\,h}\frac{\partial^2 y}{\partial t^2} + P\,\frac{\partial^2 y}{\partial x^2} = 0 \quad \text{(26)}$$

It remains to formulate the boundary conditions.

The ground motion and the method of supporting the column being the same as in the problem already solved, the boundary conditions at $x = 0$ are expressed by Equations (8*a*) and (8*b*). In the present case, however, it cannot be assumed that V and M are zero at $x = h$; instead it is necessary to write the equations of linear and angular motion for the body P. Let the values of V, M, y, and $\frac{\partial y}{\partial x}$ at the top of the column be V_1, M_1, y_1, and ϕ_1. If the body is rigid and rigidly attached to the top of the column, the value of y for its mass center is $y_1 + h'\,\phi_1$. Therefore, for the body P:

The acceleration of the mass center is $\frac{\partial^2 y_1}{\partial t^2} + h'\left(\frac{\partial^2 \phi_1}{\partial t^2}\right)$, the force causing this acceleration is $-\,V_1$; the angular acceleration is $\frac{\partial^2 \phi_1}{\partial t^2}$, and the torque about the mass center is $-\,M_1 + V_1\,h' + P\,h'\,\phi_1$, from which follow the two equations:

$$\frac{P}{g}\left(\frac{\partial^2 y_1}{\partial t^2} + h'\,\frac{\partial^2 \phi_1}{\partial t^2}\right) = -\,V_1 \quad \text{(27a)}$$

$$\frac{P\,(k')^2}{g}\,\frac{\partial^2 \phi_1}{\partial t^2} = -\,M_1 + V_1\,h' + P\,h'\,\phi_1 \quad \text{(27b)}$$

The values of V_1 and M_1 to be substituted in Equations (27) are obtained from the general formulas:

$$M = E\,I\,\frac{\partial^2 y}{\partial x^2} \quad \text{(28a)}$$

$$V = -\,\frac{\partial M}{\partial x} - P\,\frac{\partial y}{\partial x} \quad \text{(28b)}$$

the latter being equivalent to Equation (24*b*) when the terms containing W are treated as negligible. The four boundary conditions may now be written

as follows:

When $x = 0$,

$$y = a \sin\left(\frac{2\,\pi\,t}{T} + q\right) \qquad (29a)$$

when $x = 0$,

$$\frac{\partial y}{\partial x} = 0 \qquad (29b)$$

when $x = h$,

$$\frac{P}{g}\left(\frac{\partial^2 y}{\partial t^2} + h'\,\frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right)\right) = E\,I\,\frac{\partial^3 y}{\partial x^3} + P\,\frac{\partial y}{\partial x} \qquad (29c)$$

and when $x = h$,

$$P\,\frac{(k')^2}{g}\,\frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right) = -\,E\,I\left(\frac{\partial^2 y}{\partial x^2} + h'\,\frac{\partial^3 y}{\partial x^3}\right) \qquad (29d)$$

The general problem thus reduces to the solution of the differential Equation (26) subject to the boundary conditions, Equations (29). It must be remembered that, in the problem as thus formulated, the column is regarded as having inertia, but not weight; for the supported body, however, the equations take account of both inertia and weight. It is proposed first to outline the solution of this problem in its generality, and then to consider solutions based upon certain simplifying assumptions which, in some cases, may be permissible.

General Solution

For conciseness the following notation will be used:

$$\theta = h\sqrt{\frac{P}{E\,I}} \qquad (30a)$$

$$\theta' = h\sqrt{\frac{W}{E\,I}} \qquad (30b)$$

and,

$$u^2 = \frac{g\,T^2}{(4\,\pi^2\,h)} \qquad (30c)$$

Equation (26) may be written:

$$\frac{\partial^4 y}{\partial x^4} + \frac{\theta^2}{h^2}\frac{\partial^2 y}{\partial x^2} + \frac{\theta'^2}{g\,h^3}\frac{\partial^2 y}{\partial t^2} = 0 \qquad (31)$$

It is found that a solution of Equation (31) satisfying Equations (29) may be obtained by assuming,

$$y = (A_1 \cos m_1\,x + B_1 \sin m_1\,x + A_2 \cosh m_2\,x + B_2 \sinh m_2\,x) \sin\left(\frac{2\,\pi\,t}{T} + q\right) \qquad (32)$$

and properly determining the constants, m_1, m_2, A_1, B_1, A_2, and B_2. It is easily seen that Equation (32) is a solution of Equation (31) if m_1 and m_2 satisfy the

following equations:

$$m_1^4 h^4 - \theta^2 m_1^2 h^2 - \frac{(\theta')^2}{u^2} = 0 \qquad (33a)$$

$$m_2^4 h^4 + \theta^2 m_2^2 h^2 - \frac{(\theta')^2}{u^2} = 0 \qquad (33b)$$

Solving these two quadratics and noting that negative roots must be rejected because they give imaginary values of m_1 and m_2:

$$m_1^2 h^2 = \frac{1}{2}\left(\sqrt{\theta^4 + \frac{4\,(\theta')^2}{u^2}} + \theta^2\right) \qquad (34a)$$

and,

$$m_2^2 h^2 = \frac{1}{2}\left(\sqrt{\theta^4 + \frac{4\,(\theta')^2}{u^2}} - \theta^2\right) \qquad (34b)$$

By differentiation of Equation (32), expressions may be obtained for all the derivatives occurring in Equations (29). For conciseness, substitute $\frac{h'}{h} = c$, $\frac{(k')^2}{h^2} = e$, $m_1\, h = \alpha_1$, $m_2\, h = \alpha_2$, and replace $\frac{P}{E\,I}$ by $\frac{\theta^2}{h^2}$; then the resulting four equations for determining A_1, B_1, A_2, and B_2 may be written as follows:

$$A_1 + A_2 = a \qquad (35a)$$

$$\alpha_1 B_1 + \alpha_2 B_2 = 0 \qquad (35b)$$

$$\begin{aligned}&[\theta^2 \cos\alpha_1 + (u^2 \alpha_2^2 - c\,\theta^2)\,\alpha_1 \sin\alpha_1]\,A_1\\ &+ [\theta^2 \sin\alpha_1 - (u^2 \alpha_2^2 - c\,\theta^2)\,\alpha_1 \cos\alpha_1]\,B_1\\ &+ [\theta^2 \cosh\alpha_2 + (u^2 \alpha_1^2 + c\,\theta^2)\,\alpha_2 \sinh\alpha_2]\,A_2\\ &+ [\theta^2 \sinh\alpha_2 + (u^2 \alpha_1^2 + c\,\theta^2)\,\alpha_2 \cosh\alpha_2]\,B_2 = 0 \qquad (35c)\end{aligned}$$

$$\begin{aligned}&[u^2 \alpha_1^2 \cos\alpha_1 - (c\,u^2 \alpha_1^2 + e\,\theta^2)\,\alpha_1 \sin\alpha_1]\,A_1\\ &+ [u^2 \alpha_1^2 \sin\alpha_1 + (c\,u^2 \alpha_1^2 + e\,\theta^2)\,\alpha_1 \cos\alpha_1]\,B_1\\ &- [u^2 \alpha_2^2 \cosh\alpha_2 + (c\,u^2 \alpha_2^2 - e\,\theta^2)\,\alpha_2 \sinh\alpha_2]\,A_2\\ &- [u^2 \alpha_2^2 \sinh\alpha_2 + (c\,u^2 \alpha_2^2 - e\,\theta^2)\,\alpha_2 \cosh\alpha_2]\,B_2 = 0 \qquad (35d)\end{aligned}$$

Forced Oscillation of Known Period.—The coefficients of A_1, B_1, A_2, and B_2 in Equations (35) depend upon the constants of the oscillating system and also upon the period. For an oscillation of known period and amplitude the four unknowns may therefore be determined by direct solution of the four linear equations.

Free Oscillation.—The case of free oscillation also is covered by the foregoing solution as the limiting case, $a = 0$. In this case, however, u is one of the unknown quantities to be determined by Equations (35); moreover, these equations determine only the ratios of the four constants, A_1, B_1, A_2, and B_2, their actual values being indeterminate even after u becomes known. The elimination of these four quantities results in an equation involving the single unknown, u. (One method of expressing this equation is to equate to zero the determinant of the coefficients of A_1, B_1, A_2, and B_2.) Since both α_1 and α_2 depend upon u, the only practicable method of solving such an equation is to substitute

trial values of u. After u is known, any one of the four constants, A_1, B_1, A_2, and B_2, may have an arbitrary value, the other three being expressed in terms of it; this means merely that the amplitude of the free oscillation remains arbitrary.

Because of algebraic complexity the general formulas resulting from the solution of Equations (35) when u is given, are here omitted; but the procedure above outlined will be illustrated by means of an assumed set of numerical data. First, however, will be shown the form assumed by the general solution when certain simplifying assumptions are made.

Unloaded Column

If $P = 0$, the problem reduces to that solved in the main part of this paper. This case is mentioned merely to emphasize the fact that the solution of that problem is covered as a particular case by the foregoing more general solution. Thus, if $\theta = 0$, and if $m_1 = m_2 = m$, Equations (33) reduce to a form which is identical with Equation (10). If, now, in Equations (35), $\theta = 0$, $c = 0$, $e = 0$, and $\alpha_1 = \alpha_2 = m\,h$, are substituted, these equations reduce to Equations (11).

Solution When Inertia of Column Is Neglected

For a given mass element, the effect of inertia upon the oscillation of the system is greater the greater the distance of the element from the ground. There may well be cases, therefore, in which the effect of the load, P, so far predominates that the inertia of the column itself may be treated as negligible. The simplified form assumed by the solution in such a case will now be given.

The differential Equation (31) reduces to the form,

$$\frac{\partial^4 y}{\partial x^4} + \frac{\theta^2}{h^2}\frac{\partial^2 y}{\partial x^2} = 0 \quad\quad (36)$$

and a solution satisfying the boundary conditions, Equations (29), may be written in the form,

$$y = (A_1 \cos m\,x + B_1 \sin m\,x + A_2 + B_2\,x) \sin\left(\frac{2\,\pi\,t}{T} + q\right) \quad\quad (37)$$

It is easily seen that this satisfies Equation (36) if $m\,h = \theta$; and that the substitution of the proper derivatives in Equations (29) gives four equations for determining A_1, B_1, A_2, and B_2. Omitting details of algebraic transformation, the four equations may be written as follows:

$$A_1 + A_2 = a \quad\quad (38a)$$

$$B_1\,\theta + B_2\,h = 0 \quad\quad (38b)$$

$$(1 - \cos\theta + c\,\theta\sin\theta)\,A_1 + (\theta\,u^2 + \theta - \sin\theta + c\,\theta - c\,\theta\cos\theta)\,B_1 - a = 0 \quad\quad (38c)$$

and,

$$((c\,\theta\sin\theta - \cos\theta)\,u^2 + e\,\theta\sin\theta)\,A_1 - ((c\,\theta\cos\theta + \sin\theta)\,u^2 - e\,\theta + e\,\theta\cos\theta)\,B_1 = 0 \quad\quad (38d)$$

Equations (38) correspond to Equations (35) of the general solution; but in Equations (38) the solution for the unknown quantities has been partly carried out by the elimination of A_2 and B_2 from Equations (38*c*) and (38*d*).

Forced Oscillation.—If a is not 0, Equations (38) may be solved directly for A_1, B_1, A_2, and B_2, so that all the constants in Equation (37) become known in terms of a and u. This completes the solution for a forced oscillation of given amplitude and period.

Free Oscillation.—If $a = 0$, Equations (38) can be satisfied only for restricted values of u. In this case u must satisfy the following equation, obtained from Equations (38*c*) and (38*d*) by eliminating A_1 and B_1:

$$\theta\,(c\,\theta \sin\theta - \cos\theta)\,u^4 + (\sin\theta - \theta\cos\theta + (c + c^2 + e)\,\theta^2 \sin\theta)\,u^2 + e\,\theta\,(\theta\sin\theta - 2 + 2\cos\theta) = 0 \qquad (39)$$

An example with numerical data, illustrating this case, will be given subsequently.

Solution When Gravity Effect Is Wholly Neglected

In the foregoing theory both the inertia effect and the gravity effect of the load P have been taken into account, although the gravity effect of the column itself has been treated as negligible. In order to estimate the relative importance of the two effects for the load P, it will be useful to give a solution in which the gravity effect is neglected. This effect for the load P is represented by the term containing θ in Equation (31) and by the last term in Equation (36). Consider the result of omitting these terms, still treating the inertia of the column as negligible.

The differential equation becomes,

$$\frac{\partial^4 y}{\partial x^4} = 0 \qquad (40)$$

The boundary conditions at $x = 0$ are unchanged, and the conditions at $x = h$ are still expressed by Equations (27), omitting the last term of Equation (27*b*); the value of V_1 to be substituted in these equations must be determined from the relation $V = -\dfrac{\partial M}{\partial x}$. This affects only Equation (29*c*), from which the term containing P in the second member must be omitted.

The appropriate solution is expressed by the following equations:

$$y = (A_0 + A_1\,x + A_2\,x^2 + A_3\,x^3)\sin\left(\frac{2\,\pi\,t}{T} + q\right) \qquad (41)$$

$$A_0 = a \qquad (42a)$$

$$A_1 = 0 \qquad (42b)$$

$$(1 + 2\,c)\,\theta^2 A_2 + (6\,u^2 + (1 + 3\,c)\,\theta^2)\,A_3\,h + \frac{\theta^2 a}{h^2} = 0 \qquad (42c)$$

and,

$$2\,(u^2 - e\,\theta^2)\,A_2 + 3\,(2\,(1 + c)\,u^2 - e\,\theta^2)\,A_3\,h = 0 \qquad (42d)$$

Forced Oscillation.—When a is not zero, Equations (42) determine A_0, A_1, A_2, and A_3 in terms of a, u, and the constants of the system, thus completing the solution for a forced oscillation of any known period and amplitude.

Free Oscillation.—If $a = 0$, u is restricted to values satisfying the following equation, which results from the elimination of A_2 and A_3 between Equations (42c) and (42d):

$$12 \frac{u^4}{\theta^4} - 4 (1 + 3 c + 3 c^2 + 3 e) \frac{u^2}{\theta^2} + e = 0 \quad \text{(43)}$$

It is noteworthy that Equation (43) makes u directly proportional to θ, their ratio depending solely upon the constants, c and e. Remembering the meanings of u and θ, this result shows that the free period is proportional to $\sqrt{\frac{P h^3}{E I}}$. It is of interest to compare this result with Equation (14).

Solution When Supported Load Is Treated as a Particle

If the dimensions of the load are small in comparison with the length of the column, a sufficiently close approximation may be obtained by treating the load as a heavy particle. Algebraically, this amounts to assuming c and e to be zero, and results in material simplification of the formulas obtained in the foregoing solutions. In the general case defined by Equations (30) to (35), inclusive, in which the inertia of the column is taken into account, the case of free oscillation must still be solved by trial, and the value of u will depend upon both P and W. When the inertia effect of the column is neglected, however, the formulas become very simple, Equations (39) and (43), respectively, taking the following forms:

$$(\theta \cos \theta) u^4 - (\sin \theta - \theta \cos \theta) u^2 = 0 \quad \text{(44a)}$$

and,

$$12 u^4 - 4 \theta^2 u^2 = 0 \quad \text{(44b)}$$

Disregarding the zero roots, Equation (44a) gives $u^2 = \frac{\tan \theta}{\theta} - 1$; and Equation (44b) gives $u^2 = \frac{\theta^2}{3}$. A comparison of these results shows the effect of gravity upon the period of free oscillation. That this effect increases with θ, but is relatively small for small values of θ, may be seen by developing $\tan \theta$ in a power series giving,

$$\frac{\tan \theta}{\theta} - 1 = \frac{\theta^2}{3} + \frac{2 \theta^4}{15} + \cdots \quad \text{(45)}$$

Since the definition of u (Equation (30c)) gives,

$$T = 2 \pi u \sqrt{\frac{h}{g}} \quad \text{(46)}$$

Equations (44) yield the following formulas for T_0 for the oscillating particle:

Taking account of gravity effect,

$$T_0 = 2 \pi \sqrt{\frac{\tan \theta}{\theta} - 1} \sqrt{\frac{h}{g}} \quad \text{(47a)}$$

neglecting gravity effect,

$$T_0 = \frac{2\,\pi\,\theta}{\sqrt{3}}\sqrt{\frac{h}{g}} \quad \text{.......................} (47b)$$

Flexure of Column Under Static Load

It is of interest to note that the foregoing theory includes, as a special case, the theory of flexure of a column under a static load, commonly known as Euler's theory. For a column fixed at the bottom and free at the top, sustaining a centrally applied load, P, the Euler formula for incipient flexure may be written:

$$h\sqrt{\frac{P}{E\,I}} = \frac{\pi}{2} \quad \text{.......................} (48)$$

In the foregoing oscillation theory the static case is the limiting case, $T_0 = \infty$, which, in Equation (47*a*), gives $\theta = \frac{\pi}{2}$, agreeing with the Euler formula by the definition of θ (Equation (30*a*)).

Numerical Application

Assume the following numerical data for the purpose of illustrating the application of the formulas obtained in the foregoing theory, giving special attention to the case of free oscillation.

Example 1.—Assume that $\frac{h'}{h} = c = 0.3$; $\frac{k'}{h} = 0.2$; $e = \left(\frac{k'}{h}\right)^2 = 0.04$; $h\sqrt{\frac{P}{E\,I}} = \theta = 1$; and $W = P$, so that $\theta' = \theta = 1$. These values must be substituted in Equations (34) and (35); the former gives:

$$\alpha_1{}^2 = m_1{}^2\,h^2 = \frac{1}{2}\left(\sqrt{1 + \frac{4}{u^2}} + 1\right) \quad \text{.............} (49a)$$

and,

$$\alpha_2{}^2 = m_2{}^2\,h^2 = \frac{1}{2}\left(\sqrt{1 + \frac{4}{u^2}} - 1\right) \quad \text{.............} (49b)$$

Thus, for a given value of T (which fixes u), α_1 and α_2 become known and the coefficients of A_1, B_1, A_2, and B_2 in Equations (35) may be computed; the solution is thus easily completed for a forced oscillation of any given period and amplitude. For the case of free oscillation, however, u is not given, but must have a value such that Equations (35) can be satisfied when $a = 0$; such values can be found only by trial as indicated under the heading "General Solution." The trial solution involves considerable labor and the details will be omitted. It is found that the required condition is satisfied to a close approximation by $u^2 = 2.507$. By Equation (46), this value of u gives, as a possible value of the free period, $9.95\sqrt{\frac{h}{g}}$.

Example 2.—Assuming the same values of c and e as in Example 1, let $\theta' = 0$ (that is, neglect the inertia of the column). The period of free oscillation is computed: (a) for $\theta = 1$; (b) for $\theta = 0.25$; and (c), for $\theta = 0.1$. Substitution in Equation (39) gives the following:

$$(a)\quad u^4 - 2.3032\,u^2 + 0.010828 = 0$$

$$u^2 = 2.298 \quad \text{or} \quad 0.00471$$

$$T_0 = 9.53\sqrt{\frac{h}{g}} \quad \text{or} \quad 0.431\sqrt{\frac{h}{g}}$$

$$(b)\quad u^4 - 0.04977\,u^2 + 0.00001364 = 0$$

$$u^2 = 0.04949 \quad \text{or} \quad 0.0002757$$

$$T_0 = 1.398\sqrt{\frac{h}{g}} \quad \text{or} \quad 0.1043\sqrt{\frac{h}{g}}$$

$$(c)\quad u^4 - 0.007684\,u^2 + 0.000000335773 = 0$$

$$u^2 = 0.007640 \quad \text{or} \quad 0.00004395$$

$$T_0 = 0.549\sqrt{\frac{h}{g}} \quad \text{or} \quad 0.0417\sqrt{\frac{h}{g}}$$

Example 3.—With the same data as in Example 2, let the gravity effect of P be neglected. The value of u for free oscillation is now determined from Equation (43) which becomes, $12\,\frac{u^4}{\theta^4} - 9.16\,\frac{u^2}{\theta^2} + 0.04 = 0$, giving $\left(\frac{u}{\theta}\right)^2 = 0.7589$ or 0.004392; and for the three specified values of θ,

$$(a)\quad T_0 = 5.473\sqrt{\frac{h}{g}}, \quad \text{or} \quad 0.416\sqrt{\frac{h}{g}}$$

$$(b)\quad T_0 = 1.368\sqrt{\frac{h}{g}}, \quad \text{or} \quad 0.1041\sqrt{\frac{h}{g}}$$

$$(c)\quad T_0 = 0.547\sqrt{\frac{h}{g}}, \quad \text{or} \quad 0.0416\sqrt{\frac{h}{g}}$$

Example 4.—With each of the three values of θ used in Examples 2 and 3, determine the free period on the assumption that P is a particle. The simplified formulas, Equations (44) to (47), give the following results:

If gravity effect is taken into account,

$$(a)\quad \text{For } \theta = 1, \qquad u^2 = 0.5574, \qquad T_0 = 4.691\sqrt{\frac{h}{g}};$$

$$(b)\quad \text{For } \theta = 0.25, \quad u^2 = 0.02137, \qquad T_0 = 0.9186\sqrt{\frac{h}{g}};$$

$$(c)\quad \text{For } \theta = 0.1, \quad u^2 = 0.003347, \quad T_0 = 0.3635\sqrt{\frac{h}{g}}.$$

If gravity effect is neglected,

$$(a)\ \text{For } \theta = 1, \qquad u^2 = 0.3333, \qquad T_0 = 3.628 \sqrt{\frac{h}{g}};$$

$$(b)\ \text{For } \theta = 0.25, \quad u^2 = 0.02083, \qquad T_0 = 0.9067 \sqrt{\frac{h}{g}};$$

$$(c)\ \text{For } \theta = 0.1 \qquad u^2 = 0.003333, \quad T_0 = 0.3628 \sqrt{\frac{h}{g}}.$$

Summary of Computed Results.—The results computed in Examples 2, 3, and 4 are summarized in Table 1, in which the tabulated numbers are proportional

TABLE 1.—VALUES OF $2\pi u$ IN EQUATION (46)

θ	$c = 0.3$, $e = 0.04$		$c = 0$, $e = 0$	
	With gravity	Without gravity	With gravity	Without gravity
1	9.53 0.431	5.47 0.416	4.69	3.63
0.25	1.398 0.1043	1.368 0.1041	0.919	0.907
0.1	0.549 0.0417	0.547 0.0416	0.364	0.363

to the values of the free period for the several cases. The tabulation brings out clearly the fact that the importance of the gravity effect decreases as θ decreases.

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PAPERS

SIMPLIFIED WIND-STRESS ANALYSIS OF TALL BUILDINGS

BY OTTO GOTTSCHALK,[1] ESQ.

SYNOPSIS

The simple method of analysis explained herein is based on experiments to determine visible model deformations[2] and on studies leading to a purely geometrical interpretation of them.[3] The subject has been outlined by the writer several times since 1934[4] in computing inflection points under vertical loads. A structural frame is analyzed by subjecting it to given displacements. The resulting deformations are multiplied by the given loads in order to obtain the stresses.[5] This process of interpreting the model experiment directly avoids the need of solving innumerable abstract equations, by introducing only the simple expressions for the deformation curves. For one-story portal frames, with relatively stiff beams and fixed or hinged column supports, simple formulas were developed by the writer in 1932.[6] The method presented herein applies to frames of any number of stories with any number of bays and with beams and columns having any constant section.

NOTATION

The letter symbols used in this paper are defined herein when first introduced in the text. They are consistent with those introduced in a prior paper[3] and conform essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[7] compiled by a committee of the American Standards Association, with Society representation and approved by the Association in 1932.

NOTE.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **February 15, 1939.**

[1] Buenos Aires, Argentine Republic.

[2] *Journal*, Franklin Inst., July, 1926, and February, 1929.

[3] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1019.

[4] *Civil Engineering*, May, 1934, p. 264, and February, 1935, p. 101; see, also, *Civil Engineers' Digest*, 1935, No. 1, pp. 1 and 4.

[5] *Engineering News-Record*, February 27, 1936, p. 325.

[6] *Beton u. Eisen*, 1932, No. 16, p. 252.

[7] A. S. A.—*Z* 10*a*—1932.

Theory

In order to compute stresses due to wind, earthquake, or other lateral loads, the structures must be observed to determine the lateral displacements of the floors. The resistance of a column to that displacement (that is, the share of the total horizontal loads that it will absorb) is proportional to its stiffness multiplied by the sum of the relative resistance to rotation of all the members meeting at a joint. For lateral loads geometrical analysis may be greatly simplified because any approximation can be introduced that will affect, almost equally, all the members of the part of the frame under consideration.

Let k = the stiffness, $\frac{I}{h}$, of Column $A\,B$; I = the moment of inertia of the column, of height, h; m = a ratio of end rotations such that[8] $m = \frac{f}{f'} = \frac{0.5\,k}{S_B + k}$ and $m' = \frac{f'}{f} = \frac{0.5\,k}{S_A + k}$; S = a relative stiffness value such that the resistance to rotation of End A of a column is:

$$S = (1 - 0.5\,m)\,k \qquad (1a)$$

and, the resistance to rotation of End B, is:

$$S' = (1 - 0.5\,m')\,k \qquad (1b)$$

f = the intercept at End A of the tangent to the elastic curve at End B; thus,

$$f = \theta_A\,h \qquad (2a)$$

and,

$$f' = \theta_B\,h \qquad (2b)$$

θ = the angular rotation of the column relative to the final direction of its axis; thus θ_A is the rotation at End A (Fig. 1), with respect to Line $A\,B'$, and θ_B is the rotation at End B' with respect to Line $B'\,A$; and the height of the point of contraflexure is:

$$h_A = \frac{h\,(2 - m)}{3\,(1 - m)} \qquad (3)$$

Furthermore:[9]

$$\frac{f}{\Delta} = \frac{-\,S_B}{S_B + k} + \frac{m\,S_A}{S_A + k} \qquad (4a)$$

and,

$$\frac{f'}{\Delta} = \frac{S_A}{S_A + k} - \frac{m'\,S_B}{S_B + k} \qquad (4b)$$

and the force, P, necessary to produce Displacement $\Delta = 1$, is:

$$P = \frac{6\,E\,k\,(f' - f)}{h^2} \qquad (5)$$

[8] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1022, Equations (1) and (2).

[9] *Loc. cit.*, Equations (11), (12), and (15), pp. 1025, 1027.

In Equation (5) $\frac{6\,E}{h^2}$ is equal for all the columns of any one story. The horizontal resistance, therefore, against a unit side-sway (that is, the percentage that each column of a story absorbs of a horizontal force acting upon the floor above) is in proportion to $k\,(f' - f)$. None of the many approximate methods of wind-stress analysis of frames proposed heretofore yields safe results, unless it takes that quantity into consideration. The complexities introduced by poor proportioning of columns were emphasized by George E. Large, Assoc. M. Am. Soc. C. E., and Samuel T. Carpenter, Jun. Am. Soc. C. E., in 1934.[10]

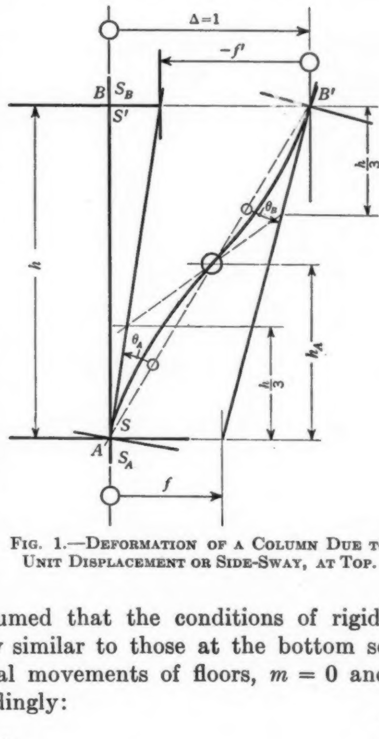


Fig. 1.—Deformation of a Column Due to Unit Displacement or Side-Sway, at Top.

Equations (4) may be considerably simplified for typical stories. In Fig. 1, when Column $A\,B$ is fixed at one end (that is, when $S_B = \infty$) $m = 0$ and $S = (1 - 0.5\,m)\,k = k$ at the other end.[3] As an opposite extreme assumption, that is, when $S_B = k$, the corresponding values are: $m = 0.25$ and $S = (1 - 0.5 \times 0.25)\,k = 0.875\,k$. The range of values of m and $\frac{S}{k}$, then, is small. For typical stories, therefore, it may safely be assumed that the conditions of rigidity at the tops of columns are sufficiently similar to those at the bottom so that, in all structural frames, for lateral movements of floors, $m = 0$ and $S = k$. Simplifying Equations (4) accordingly:

$$\frac{f}{\Delta} = \frac{-\,k_B}{k_B + k} \qquad \text{(6a)}$$

and,

$$\frac{f'}{\Delta} = \frac{k_A}{k_A + k} \qquad \text{(6b)}$$

in which k_A and k_B, instead of S_A and S_B are the sum of the stiffness, $k = \frac{I}{h}$, of the structural members joining Line $A\,B$ at Points A and B, respectively. For the purpose of the present analysis, therefore, one may read the assumed k-values directly instead of computing the value of S first.

Note that, in Equations (6), f and f' have different signs and, therefore, they must be added in Equation (5).

As a further admissible simplification, when analyzing any one typical story, the designer may disregard the columns above and below that story.

[10] *Engineering News-Record*, May 17, 1934, p. 637.

With all those simplifications, which affect the deformations of all columns in nearly equal proportion, results have proved to be quite close to those found by the "exact" abstract analysis—safely within 5% for shears and reactions, and little more for bending moments when assuming inflection points at the middle of the column heights of typical stories.

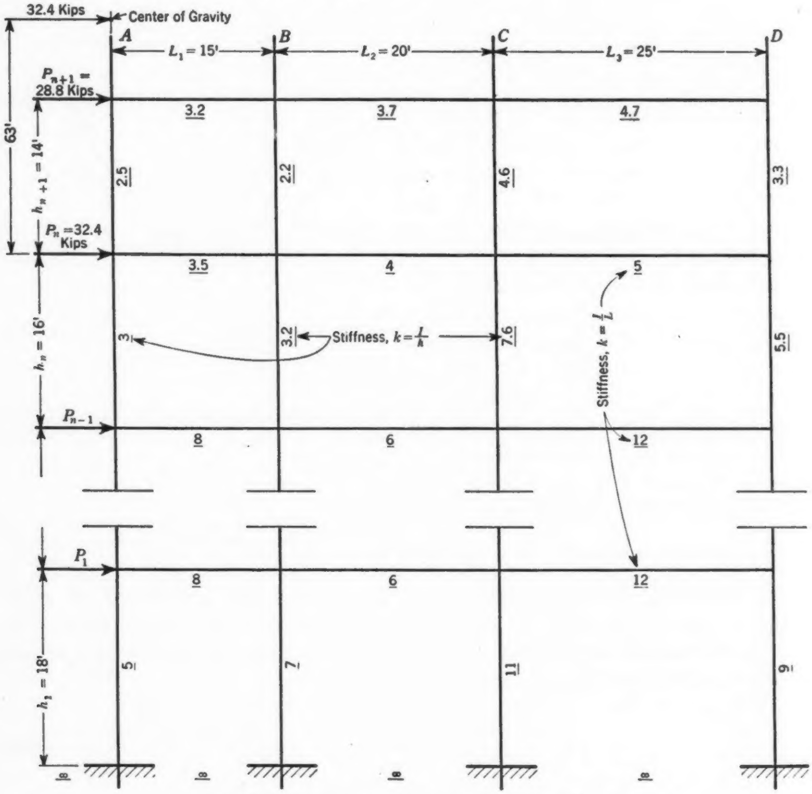


FIG. 2.—DIAGRAM OF TYPICAL STRUCTURAL FRAME

Fig. 2 is the diagram of the ground floor and two typical upper stories of a structural frame, showing the assumed k-values $\left(\frac{I}{L}\text{ in beams and }\frac{I}{h}\text{ in columns}\right)$, all of which are different. At the ground floor, on both sides of the fixed base of the interior and exterior columns $k_A = \infty$, and, therefore, $\frac{k_A}{k_A + k} = 1$. At the nth and $(n + 1)$th floor horizontal wind loads, $P_n = 32.4$ kips and $P_{n+1} = 28.8$ kips, may act. (The kip, or "kilo-pound" equals 1 000 lb.)

Each story and, within each story, each bay are next analyzed separately by subjecting the upper beams of the nth story (see Fig. 3(a)) to a side-sway,

$\Delta = 1$. Interior columns always form part of two adjacent bays, and interior beams, part of two stories. The height, h, is constant for all columns and, as the designer is only concerned with relations between the deformations of the several columns, he need simply write at every angle the $\frac{k_B}{k_B + k}$-values at the upper beams and the $\frac{k_A}{k_A + k}$-values at the lower beams. For example, at Point D at the top floor $(n + 1)$, $\frac{5}{5 + 5.5} = 0.48$, and at the next floor below (n), $\frac{12}{12 + 5.5} = 0.69$; and for Column D the sum of resistance at the nth story, $0.48 + 0.69 = 1.17$.

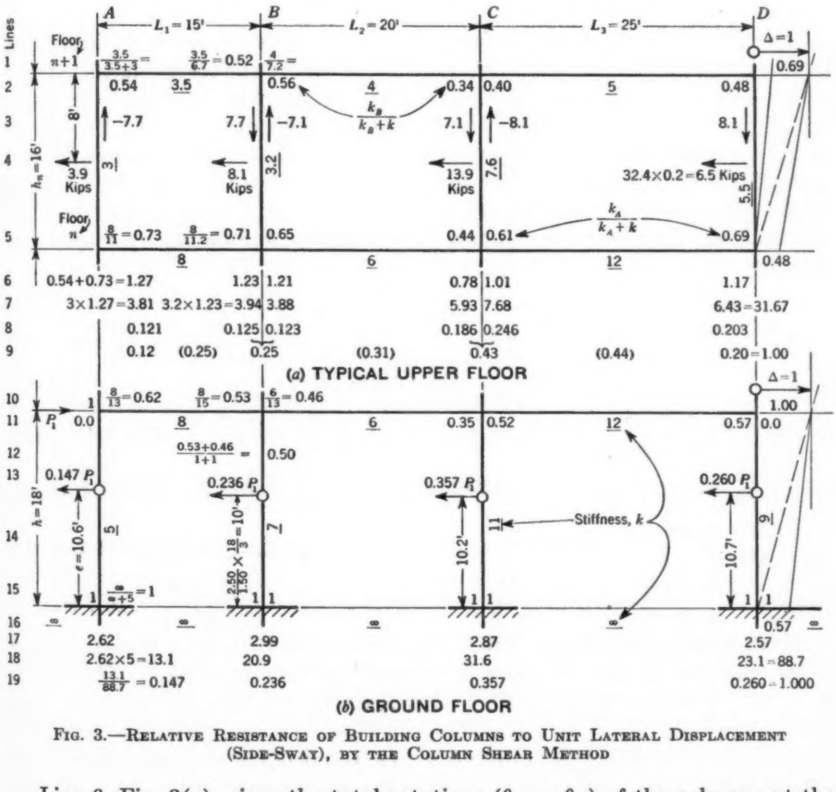


FIG. 3.—RELATIVE RESISTANCE OF BUILDING COLUMNS TO UNIT LATERAL DISPLACEMENT (SIDE-SWAY), BY THE COLUMN SHEAR METHOD

Line 6, Fig. 3(*a*), gives the total rotations $(\theta_A - \theta_B)$ of the columns at the left and at the right sides of the given bay, and Line 7 gives the values of $k(\theta_A - \theta_B)$. The sum of the values in Line 7 is 31.67 and, consequently, the relative resistance values, shown in Line 8, are $\frac{3.81}{31.67} = 0.121$; $\frac{3.94}{31.67} = 0.125$; $\frac{3.88}{31.67} = 0.123$; etc. In Line 9 the relative resistance of the columns of the nth

story to the lateral load, $P_n = 1$, at the top, equals 0.121, $0.125 + 0.123 = 0.25$, etc. Similar values for the columns of the $(n + 1)$ story, for a lateral load of $P_{n+1} = 1$ at the top, have been computed as 0.135, $0.125 + 0.140 = 0.265$, $0.198 + 0.217 = 0.415$, and 0.185.

At the ground floor (Fig. 3(*b*)) all the columns are fixed (that is, $k_A = \infty$). Therefore, $\theta_A = \frac{k_A}{k_A + k} = \frac{\infty}{\infty + k} = 1$ (Line 15, Fig. 3(*b*)). In Line 17, $(\theta_A - \theta_B) = 2.62$, 2.99, 2.87, and 2.57; and in Line 18, $k\,(\theta_A - \theta_B) = 2.62 \times 5 = 13.1$; $2.99 \times 7 = 20.9$; $2.87 \times 11 = 31.6$; and, $2.57 \times 9 = 23.1$—total 88.7. Every column then absorbs of a lateral load, $P_1 = 1$, at the top in the proportion: $\frac{13.1}{88.7} = 0.147$; $\frac{20.9}{88.7} = 0.236$; $\frac{31.6}{88.7} = 0.357$; and, $\frac{23.1}{88.7} = 0.260$ (see Line 19).

For the purpose of computing stresses the designer needs to know the location of the inflection point in the columns, where the foregoing thrusts are assumed to act. For typical stories those points may be assumed, closely enough, to act at mid-height. (Hale Sutherland and Harry L. Bowman,[11] Members, Am. Soc. C. E., propose values that are slightly different, as a result of studies of a large number of examples.) For the ground floor, with columns fixed at the bottom, conditions at the top and bottom vary greatly and the points of inflection are to be computed at a distance, h_A (see Fig. 1), from Point A, equal to:

$$h_A = \frac{h\left(2 - \frac{\theta_B}{\theta_A}\right)}{3\left(1 - \frac{\theta_B}{\theta_A}\right)} \quad \text{(7)}$$

In Fig. 3(*b*), $\frac{h}{3} = \frac{18}{3} = 6$ ft; and, at Column A: $\frac{\theta_B}{\theta_A} = \frac{-0.62}{2.00} = -0.31$. The height, h_A, to the point of contraflexure is $6 \times \frac{2.31}{1.31} = 10.6$ ft. For Column B: $\frac{-(0.53 + 0.46)}{2.00} = -0.50$ and $h_A = \frac{6 \times 2.5}{1.5} = 10$ ft. Similarly, at Column C: $h_A = \frac{6 \times 2.44}{1.44} = 10.2$ ft and, at Column D: $h_A = \frac{6 \times 2.29}{1.29} = 10.7$ ft.

Having analyzed the structure in this manner the horizontal thrust or shear in the columns in the nth floor is obtained as follows (Line 4, Fig. 3(*a*)): $T_A = 32.4 \times 0.12 = 3.9$ kips; $T_B = 32.4 \times 0.25 = 8.1$ kips; $T_C = 32.4 \times 0.43 = 13.9$ kips; and, $T_D = 32.4 \times 0.20 = 6.5$ kips. Assuming that the inflection points are at the mid-height of the columns, the bending moments at the tops will be $8.0 \times 3.9 = 31.2$ kip-ft; $8.0 \times 8.1 = 64.8$ kip-ft; $8.0 \times 13.9 = 111.2$ kip-ft; and, $8.0 \times 6.5 = 52.0$ kip-ft, respectively.

The vertical reactions in the columns of the nth floor, produced by the wind load at the nth floor, may be obtained, assuming that the several bays of the story must resist the overturning wind moment of $28.8 \times 7.0 + 32.4 \times 8.0 = 460.8$ kip-ft. As indicated in parentheses in Line 9, Fig. 3(*a*), the part of the

[11] "Structural Theory and Design," by Hale Sutherland and Harry Lake Bowman, N. Y., John Wiley & Sons, Inc., 1930, Method C, p. 207.

horizontal thrust absorbed by the three bays, respectively, is 0.25, 0.31, and 0.44; and, consequently, the shear within the bays is $\frac{0.25 \times 460.8}{15} = 7.7$ kips; $\frac{0.31 \times 460.8}{20} = 7.1$ kips; and, $\frac{0.44 \times 460.8}{25} = 8.1$ kips. The reaction at each of the four columns is: $R_A = -7.7$ kips; $R_B = 7.7 - 7.1 = 0.6$ kips; $R_C = 7.1 - 8.1 = -1.0$ kips; and, $R_D = 8.1$ kips.

When only column reactions or beam moments are wanted it may be preferable and sufficiently accurate to analyze the beams directly for shear as demonstrated in Fig. 4. The top beam of the nth story is assumed to be

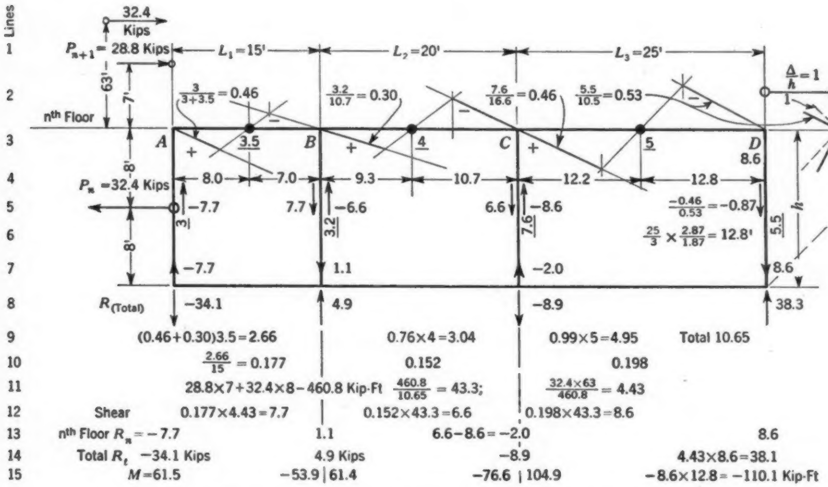


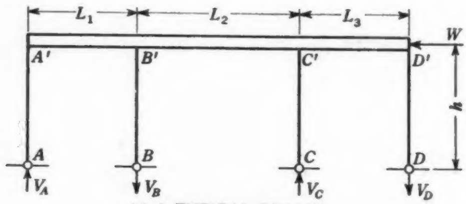
FIG. 4.—APPROXIMATE ANALYSIS TO DETERMINE SHEAR IN BEAMS AND REACTIONS, R, IN COLUMNS, BY THE BEAM SHEAR METHOD

deflected laterally a distance, Δ, assuming for convenience that $\Delta = h$, with respect to the beam below. The approximate relative rotations at the four columns, are thus determined as shown. In Line 2, Fig. 4: $\theta_A = 0.46$; $\theta_B = 0.30$; $\theta_C = 0.46$; and, $\theta_D = 0.53$. These values may be assumed positive on the right and, therefore, negative on the left of the columns. Note that those rotations are only relative values and must not be used separately for the analysis of any beam.

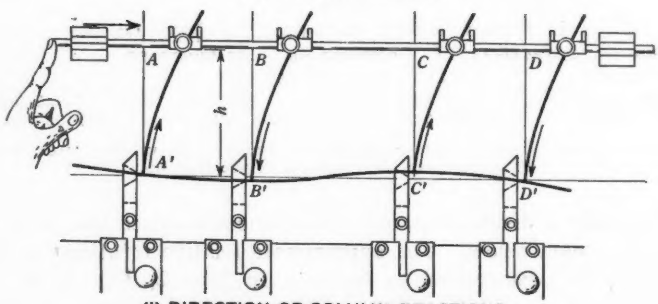
Referring again to Equation (5), Line 10, Fig. 4, gives, for the three beams, $A\,B$, $B\,C$, and $C\,D$, the coefficients of shears, $\frac{(f - f')\,k}{L^2}$, as 0.177, 0.152, and 0.198, respectively; and, in Line 9, the same values are multiplied by the several span lengths to yield a total of 10.65. Therefore, Line 9 represents the sum of the moments of the shear at the ends of the beams, equal to the moment of wind loads with respect to the Axis $A\,B\,C\,D$, which (as computed previously and entered in Line 11, Fig. 4) is 460.8 kip-ft. Since $\frac{460.8}{10.65} = 43.3$, the shear

in the several openings is, as indicated in Line 12: 43.3 × 0.177 = 7.7; 43.3 × 0.152 = 6.6; and, 43.3 × 0.198 = 8.6 kips.

The reactions at Columns A, B, C, and D, corresponding to the nth floor, are as shown in Line 13, Fig. 4, as follows: $R_n = -7.7$; $7.7 - 6.6 = 1.1$; $6.6 - 8.6 = -2.0$; and, 8.6 kips, respectively. These reactions would have to be computed in a similar manner for all the floors above, and added from the top floor down to the nth floor to obtain the total column pressure, but generally it will be close enough to increase the reactions obtained for the nth floor in relation to total moments, as follows: Suppose that the center of gravity of the wind load (32.4 kips) at the nth floor is at 63 ft above the nth floor, as shown in Fig. 2; then the reactions, R_n, obtained previously (see Line 13, Fig. 4) must be multiplied by $\frac{32.4 \times 63}{460.8} = 4.43$, making the total reactions, R_t, in the columns of the nth floor, due to wind pressure (see Line 14) $= 4.43 \times 7.7 = -34.1$; 4.9; -8.9; and, 38.1 kips.



(a) A TYPICAL FRAME



(b) DIRECTION OF COLUMN REACTIONS VISUALIZED BY EXPERIMENT

FIG. 5.—EXPERIMENT VISUALIZING THE DIRECTION OF COLUMN REACTIONS

The inflection points at distances, h_A, from the left and at h_A' from the right ends of the beams produced by the side-sway in Fig. 4, have been computed by Equation (3). Multiplying the shears, Line 12, Fig. 4, by these values of h_A and h_A', gives the bending moments at the several ends of beams produced by wind load at the nth floor, as given in Line 15, Fig. 4.

It will be noted that the reactions in the columns are generally positive and negative, alternately, and not (as frequently assumed in wind-stress analysis) positive at one side of the center of the frame and negative on the other side. Such arbitrary assumptions may lead to considerable error,

especially when the lengthening and shortening of columns is being taken into consideration.

The simple experiment in Fig. 5 will illustrate how the signs for column pressures may alternate. Fig. 5(*a*) shows a typical portal frame and Fig. 5(*b*) is the model reproducing it, held upside down at the top of the columns. The column bases are held at their original distance from each other by means of clamps fixed on a sliding rod. When the rod is pushed to the right the first and the third columns of the model are being pulled upward and the second and fourth columns are pushed down, indicating pull and compression in alternate columns, respectively.

For very tall buildings the secondary stresses from the shortening of compressed columns and the lengthening of expanded columns, may be of interest. Suppose that there is a possible average of 6 000 lb per sq in. computed stress due to wind loads alone in exterior steel columns (modulus of elasticity = 30×10^6), the column would shorten or lengthen $\frac{2}{10.000}$ or, 0.24 in. in every 100 ft. Eventually, part of this deformation will be absorbed by the masonry.

When designing very tall frame structures, therefore, it may be convenient to compute column stresses by either one of the methods shown in Figs. 3 and 4 and correct the column loads according to the change of length to be expected, applying Fig. 1 and Equation (5). As an example, assume that, at a great height, in a tall building the leeward column shortens 1 in. with respect to the nearest interior columns; and, assume a restraint equivalent to 50% fixation at both ends of the adjoining beam: $f - f' = 1$ in. If that beam has a length of $L = 200$ in. and a moment of inertia of $I = 1\,000$ in.^{4}, making $k = \frac{1\,000}{200}$ $= 5$ in.^{3}, Equation (5) yields:

$$P = 6 \times 30\,000\,000 \times 5 \times \frac{1}{200^2} = 22\,500 \text{ lb} = 22.5 \text{ kips}$$

which is the amount by which the load on the exterior column must be decreased and the load on the interior column increased for a 1-in. difference of level.

Conclusion

The elementary simple methods, derived from the visible model experiment, which are presented for immediate application in Figs. 3 and 4, should prove a convenient tool in structural design. By merely starting from actual $k\left(=\frac{I}{L}\right)$-values of any frame the designer can take into account the practical peculiarities encountered in the field. That is not possible with prior methods of approximation, nor with the abstract mathematical treatments, except, perhaps, for the simplest of structures. Fig. 3 especially presents the only procedure known to the writer of analyzing, conveniently, those frequent cases of application in the field, where the stiffness value, $k = \frac{I}{L}$, of some of the sus-

taining members of a skeleton is relatively infinite, such as reinforced concrete stair-cases and elevator shafts.

The "column shear method," demonstrated in Fig. 3, may be applied to advantage generally. The "beam shear method" in Fig. 4 requires even less work and is recommended for quick calculation of reactions in columns subjected to wind pressure and for revising the design of structural frames to take account of possible differences of levels produced in tall columns by horizontal loads, impacts, or unequal changes of temperature.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

TRAFFIC PROBLEMS IN METROPOLITAN AREAS

By Earl J. Reeder,[1] Esq.

Synopsis

The purpose of street traffic is to convey persons from where they are to where they want to be, and goods from where they are to where they are wanted. Time is an important factor because delays defeat this purpose in part. Safety is an even more important factor because accidents defeat the purpose entirely.

In general, the traffic problems before all municipalities to-day relate to safeguarding the purpose of traffic by reducing the probability of congestion and accidents and by helping traffic to move expeditiously and safely. In metropolitan areas these problems are generally more serious than in smaller communities because the concentration of traffic is greater. Although these problems may be variously classified, they will be treated herein under six headings, some of which overlap to some degree. These headings are: (1) Helping traffic through danger spots; (2) protecting the person on foot; (3) stopping the increase in night accidents; (4) reducing traffic concentration; (5) keeping speeds safe for conditions; and (6) providing for loading and parking.

Helping Traffic Through Danger Spots

Contrary to the belief of many, a city street is not a continuous scene of human and mechanical disaster. In fact, traffic runs along quite well at most places, thanks to the judgment and skill of the majority of drivers and pedestrians. At certain points, however, accidents and congestion problems tend to recur because the difficulties are more serious than individuals can cope with successfully.

In general, a higher concentration of accidents occurs in the more congested districts, although the severity is greater in areas with less congestion. Little trouble is encountered where the movements are simple and clearly evident to

Note.—This paper was presented at the meeting of the Highway Division, New York, N. Y., January 20, 1938. Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **February 15, 1939.**

[1] Chf. Traffic Engr., National Safety Council, Chicago, Ill.

drivers and pedestrians. Where the movements are complex and confusing, drivers and pedestrians do not fully understand the situations before them and do not have time to analyze these problems for themselves.

Proper accident records are imperative for determining where these "sore spots" are, and for deciding upon measures which will solve the problems as they exist. There is no better single indication of what is wrong with traffic than the accidents that occur in it. In fact, there is no indication that equals the accident records in importance. Traffic volumes are important in the solution of traffic problems, but they do not show where the conflicts are. Speed observations are helpful but, likewise, they do not show where traffic gets into difficulties. The accident records show where, when, and how these conflicts occur. Any community which does not have good accident records should provide for their proper accumulation as the first step in attacking the traffic problem.

One important problem of metropolitan areas (important there because traffic is denser than in smaller municipalities) is to simplify traffic at the "sore spots" and to provide assistance for vehicles and pedestrians to move safely. The purpose of the signs, signals, and markings which are installed for traffic control is to give the individual driver or pedestrian a little greater assurance that others are going to move in a certain way at a certain time so that he may act accordingly. For example, a "stop" sign at an intersection is intended to bring traffic from one direction to a full stop so that the drivers in that direction will work their way into the traffic on the cross street at moderate speeds after finding out that the way is clear.

One of the important principles in traffic control is to provide enough restriction or assistance, but no more than is necessary for safe and expeditious movement of traffic, because, after all, human beings do desire to be safe and will act in accordance when they know what to do. Acting on this principle, the time is passing when a "stop" sign, a "stop-and-go" signal, or some other favorite measure is expected to prevent all kinds of accidents and eliminate congestion from any cause. Studies of accidents before and after the application of these various measures have shown that unreasonable regulations—restrictions which are not adapted to the accident experience pattern, for example—do not accomplish results. There is a direct relation between stop-sign obedience and the reduction in speed that is necessary for safety, determined in terms of locations, of view obstructions on corners, street widths, and other definable factors. As a result of these studies, the stated speed "slow" sign which reads, "Slow to 11 Miles," "Slow to 14 Miles," or slow to any other speed which is computed as the critical speed, has been developed for use where full stops are not necessary for safety, but some restriction of speed is needed. This sign slows down the driver to the speed at which he can determine what the other drivers are doing and can act accordingly.

The development in "stop-and-go" signals and in their use is another good example of the simplification of traffic movement to avoid conflicts. Studies made in a number of cities in 1930–31 showed that such signals are not panaceas for all traffic ills but are invaluable for the types of problems that they can solve. Gradually, their adaptation to complicated places has

been expanded. More phases of the cycle of colors have been added for complicated situations. Pedestrian "walk" lights have been developed. Special "turn" arrows have been added for unusual movements. Traffic and pedestrian actuation has been developed for fitting the operation more nearly to the traffic demand.

In metropolitan areas the demands upon traffic control reach their highest degree of concentration. It is there that greatest complications of traffic occur; and it is there also that the efficient use of the streets at the same time that the greatest of safety is provided, is imperative. For that reason extensive inter-connected signal systems have become necessary to enable vehicles to move with uniformity and continuity and, at the same time, to provide protection for pedestrians. Although such systems must often be complicated as to design, they must be simple so far as their appearance to the public is concerned because their purpose is to simplify traffic movement and make it easier for each driver and pedestrian to understand what he is supposed to do at each point.

No signal installation meets this condition more fully than the type in use on Michigan Avenue, in Chicago, Ill. The timing is extremely complicated because, at points, traffic is controlled independently for opposite sides of the street. However, every effort has been made to make clear, to drivers and pedestrians, how they are supposed to use the signals. Turning signals show when certain turns may be made to avoid conflict with other movements. Pedestrian signals show when pedestrians may cross. Traffic is kept in step by flashing the part of the green light that is not normally required for handling the volume of traffic at the less important intersections.

To simplify traffic by signs, signals, and markings, the engineer must do some more or less complicated planning in order to determine what should be done at certain locations and he must give the public the advantage of this solution by unmistakable means. Careful engineering planning is thus substituted for snap driver judgment, to a large extent. Traffic control systems of the future will probably be considerably more complicated in design than those of the present, although traffic movement will be greatly simplified thereby. As public understanding increases through traffic safety propaganda, public acceptance of such aids will be greater, and the results will be even more successful than those that are now being experienced from such measures.

Protecting the Person on Foot

The pedestrian problem is one of increasing seriousness. The greater the speed and mobility of the vehicle, the more difficult it is for the pedestrian to foresee the hazards that surround him. The freedom of pedestrian action of former days must be restricted as the differential between vehicle speeds and pedestrian speeds increases. This is particularly true in congested areas where there are more pedestrians as well as more vehicles.

The pedestrian problem is particularly serious among persons who have never driven automobiles and who do not appreciate the limitations upon vehicle performance. Perhaps this fact is best shown by data from Connecticut for a recent period of five years during which 1 231 pedestrians were killed.

Of these, 83% were of sufficient age to be eligible for drivers' licenses, but only 5% were licensed operators. This emphasizes more effectively than almost any other available data the importance of public education, as well as actual control measures, for preventing accidents.

In the past, too little attention has been devoted to the pedestrian, in devising control measures. Too little emphasis has been placed upon the need for educating the pedestrian concerning the hazards of traffic. Propaganda has been directed at the driver and control measures have accrued to his benefit; but the pedestrian has been considered a free agent who can protect himself if he is sufficiently concerned.

There are, and will continue to be, many "stop-and-go" signals without special lights for the pedestrian; and he must be brought to closer conformance with the usual signal indications. To encourage this co-operation the cycle of operation of signals should not be longer than is necessary to handle the traffic efficiently because pedestrians will not wait when there is no traffic moving.

The pedestrian education program must often be selective. In a recent traffic survey of Vancouver,[2] B. C., Canada, it was found that the residences of many of the pedestrians, as well as of many of the drivers, who were involved in accidents, were in areas of special racial, language, or social characteristics which would not commonly be reached by the ordinary means of public propaganda. Many of them lived in the Oriental sections, for example. The usual clubs, English language newspapers, and similar agencies did not reach those people, and their lack of appreciation of the traffic problem made many of them unsafe drivers and unsafe pedestrians. The public must be educated concerning traffic problems through agencies that they understand and in which they have confidence.

Stopping the Increase in Night Accidents

Closely related to this problem of pedestrian protection is that of the increasing seriousness of night accidents. Pedestrians, in particular, are susceptible to accidents at night because they carry no lights, and dark clothing often gives little or no reflection of vehicle lights. For their protection proper street lighting is a necessity. Experiences in Detroit, Mich., Evanston, Ill., and other localities have shown that good lighting does reduce certain kinds of night accidents, particularly those involving pedestrians.

How far the engineer can hope to go in providing lights for rural highways may be a problem involving some controversy, but concerning the proper lighting of city streets where vehicles and pedestrians must intermingle—and street intersections are frequent—there can be no question. It is there that the problems of night accidents are more highly concentrated. It is there that headlights often are of least value in providing adequate visibility.

Adequate lighting is more than a question merely of increasing the candle power of the luminaries. It involves proper spacing, mounting height, position with respect to the street, and types of luminaries. It is more than mounting

[2] "Improving Street Traffic, Vancouver, B. C.," by the National Safety Council, 1936–38.

a light on top of a post to show approximately where the street is. It is a matter of providing illumination to reveal objects that would otherwise be hidden from the view of drivers at night.

Here, again, the best indication of what is wrong is the accident records. In the traffic survey of Honolulu, Hawaii, in 1936, a spot map of night accidents was prepared and it was found that the spots were particularly dense at certain points and along certain streets, and that they were very few at other places. In general, the density of the spots seemed to bear an inverse relation to the quantity of the lighting, much of which was very defective. Obviously, the proper approach was to improve the lighting first where the ratio of night accidents to day accidents was the greatest. That is a fair approach for any city to make to its street-lighting problem.

The night-accident problem is not to be attacked entirely from the standpoint of general lighting although that is an important part. Special hazards must be specially lighted. Many persons have been killed or injured in vehicle collisions within improperly lighted safety zones, for example. Center-posts in railway underpasses have taken their toll. Signs that were not visible at night have failed to prevent those accidents which they prevented in the day time. The traffic problems at night are entirely different from daylight problems because the very source of illumination is different. Special illumination is required at hazardous places to make sure that the hazards can be seen, and that the way to avoid them is apparent.

Reducing Traffic Concentration

One of the fallacies that must be exposed and overcome is the feeling that is often held by merchants and business people that traffic density is a good sign of business prosperity. It may, rather, be a sign of impending business decay and decentralization. An important problem is to differentiate between potential business traffic and through traffic that has no commercial significance. In some cities as much as 30% of the traffic in central business districts passes through without making any business stops. Means should be provided for traffic that is not potentially commercial to by-pass congested districts so that these areas may be reserved for easier access by drivers and pedestrians who go there expressly for business purposes.

This is to be done by providing convenient routes properly marked and safeguarded. It is not to be done by compulsion. Traffic that is not potential business can be directed easily over any convenient route that is properly marked and is a more comfortable route to travel than the streets through the central business district.

Too many cities are trying simultaneously to maintain convenient access to the business houses for business purposes and to handle considerable through traffic which can do nothing but make it more difficult for potential customers and clients to patronize the business places. Persons who are not potential buyers prefer to use routes that do not have the confusion, congestion, and delays of business streets. However, traffic tends to go toward business districts if alternate routes are not properly marked because it is in such districts

that drivers can get their direction to their destination most conveniently; and the business districts are usually plainly seen from almost any part of the city. It is only through proper routing that the necessary segregation of business and non-business traffic can be made.

Keeping Speeds Safe for Conditions

One of the big problems of municipalities is to adapt traffic speeds to conditions. The day of installing speed-limit signs at the boundaries of municipalities and leaving the individual driver to determine whether he is driving safely as long as he keeps within these speeds, is passing rapidly. The obligation of speed regulation for safety is not discharged that easily. Particularly on densely-traveled arterial routes, the problem of directing the traffic means moving it as rapidly as the physical conditions and safeguards will permit in safety. It is not always possible for the driver to judge accurately the maximum safe speed from place to place because hazards are sometimes hidden. Speed zoning, in which the maximum permissible speeds are determined by engineering investigation and are posted conspicuously, is an important new development. It is almost as necessary as route-marking for directing traffic conveniently and expeditiously, with safety.

One of the most important traffic problems is to create an adequate public understanding of the meaning of speed zoning, so that drivers will observe the restrictions that are imposed. They are likely to look upon any speed limit as a remnant of former days when the many who had no automobiles did not hesitate to impose extreme restrictions upon the few who did have them. Traffic administrators must show these drivers that the purpose of a speed-zoning sign is to give every driver the advantage of the detailed engineering investigation, which may have taken days or weeks in order to determine the critical speed so that he may not have to depend upon his momentary judgment in an emergency to determine how fast he can go.

In "stop-and-go" signal systems, proper inter-connection and timing are very effective in speed control. Progressive signal systems are planned to allow vehicles to pass through in platoons at pre-determined speeds. If they go at appreciably higher or lower speeds, they come to red lights and are stopped. Such designs are controlled by the volume of traffic that must be handled, the block lengths, and the cross-traffic at the different intersections. There probably is no more intriguing problem in traffic engineering than the design of a flexible progressive control system for a street with varying block lengths and dense traffic. Regardless of their actual complications, their indications to the public must be simple, direct, and unmistakable, as well as reasonable.

Speeds must be appropriate for street conditions. Where drivers are unable to recognize, quickly, the practical limitations on their speed, restrictions must be applied. As in traffic control for all other purposes, these restrictions must be reasonable and they must be adequately "sold" to the public. Most drivers will comply with such restrictions when they are understood, but some must be required to do so by enforcement.

Providing for Loading and Parking

Parking is a problem involving much controversy in congested business districts. It is probably one of the most misunderstood problems in the entire field of traffic administration. There are those who want unlimited parking and those who want to eliminate it altogether, depending upon their respective interests in the problem.

The primary purpose of parking, of course, is to provide access to business houses in business districts. It is a problem which usually develops simultaneously with other problems. The denser the traffic, the more acute is the parking demand likely to be and the more serious is the conflict.

The primary purpose of providing for access to the curb is to receive or discharge passengers or merchandise. When this has been provided to the full extent that is required, the remaining free curb space may be used for parking. However, the process has often been reversed—the curb has been parked full of cars in actual storage for various lengths of time, and so the driver, in order merely to load or unload, has been forced to take any place he could find to stand his vehicle for a few minutes. It may have been opposite a fire hydrant or a safety zone, or double-parked in the street, if no place at the curb was available.

It is inconceivable that cars will continue to be permitted to park almost bumper to bumper in busy city streets, making it difficult for vehicles to load or unload at the curb. More off-street parking and loading spaces must be provided. The time will soon come when restrictions will require all business buildings to have off-street provisions for parking and places within them, or on adjacent property, for the loading or unloading of merchandise, leaving the near-by streets free for stops to load or unload passengers.

Until that day cities must resort to time restrictions upon parking which will insure a sufficiently rapid turnover in the use of curb space to give every one a chance and to provide for convenient loading and unloading. A new development in the supervision of parking regulations is the parking meter which shows, mechanically, the length of time that has elapsed since a coin was inserted by the driver who parked there and reveals at once the expiration of that time and the act of over-parking. Meters do not solve the parking problem, but they do help the police materially in supervising and enforcing the restrictions that exist.

Mass Transportation

In dealing with these problems of traffic one must not overlook mass transportation. In the street space used per person carried, the private passenger vehicle is much less efficient than the bus or street car. In the placing of street car and bus stops, in the establishment of routes, and even in the restriction of traffic, buses and street cars should be impeded the least that is consistent with safe and expeditious movement of all traffic.

Many of the measures that have been mentioned apply as well to mass transportation as to private passenger vehicles. Others must be adapted. The timing of "stop-and-go" signals applies to both. The establishment of

bus stops requires the special clearing of parked vehicles so that the stops may be made at the curb in the most convenient place for all concerned. Like all other problems in traffic regulation and control, those pertaining to mass transportation must be solved on the basis of the conditions that exist. Accident records must be consulted, traffic volumes must be considered, and the demand for service must be taken into account and properly evaluated.

Conclusion

Traffic planning is distinctly an engineering function. The day is long past when trial-and-error methods are appropriate for solving the traffic problem. In former days traffic officials had to learn from experience by trying something and, if it did not work, trying something else. From that experience they have learned many things about the effectiveness of different measures for solving many kinds of problems.

Engineers still face the necessity for testing their planning efforts against actual results in accidents prevented, congestion reduced, and traffic expedited. Every measure that is applied should be preceded by the best analyses that can be made of the important facts about the problem to determine what is the most reasonable measure to be applied. It should be followed by equally thorough analyses of the results, to decide whether accidents have been prevented and, if so, what kinds. The engineer must determine whether traffic moves more freely and whether over-all time between destinations has been reduced. He must know whether he is handling more traffic or less, because it has been driven to some other streets. All these facts help him to guide his next steps in traffic planning, to apply appropriate corrective measures, and to anticipate the needs of traffic when he builds new streets or makes improvements.

Traffic planning provides the facilities for traffic movement. It provides the aids and safeguards that are needed. It attempts to determine what normal drivers and pedestrians do in traffic and to fit the facilities and regulations accordingly. Traffic planning must be reinforced by adequate education to inform the public about the use of the facilities that are provided. It must also be supported by adequate enforcement to require obedience from those who are wilful or negligent in their disobedience or misuse of the facilities. However, the responsibility of the engineer in traffic planning is to adapt his measures as nearly as he can to normal driver and pedestrian practices

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DISCUSSIONS

ECONOMIC PIPE SIZES FOR WATER DISTRIBUTION SYSTEMS

Discussion

BY THOMAS R. CAMP, M. AM. SOC. C. E.

THOMAS R. CAMP,[19] M. AM. SOC. C. E. (by letter).[19a]—In the preparation of this paper, it was the writer's purpose to contribute in a small way toward the development of effective and reliable means for the design of water distribution systems and reinforcing mains. As is well known, the analysis of flow in distribution systems is extremely complicated. Since reliable methods for the solution of the many hydraulic problems relating to distribution networks have been made available only recently, it is small wonder that so little has been done regarding the rational approach to methods of selecting the most economical sizes of pipes. Nevertheless, the selection of economic pipe sizes is of as much importance to the engineer in the design of distribution systems as is the correct solution of the hydraulic problems.

A most necessary part of engineering design is the selection of sizes and shapes for best economy. In the design of most engineering works, considerable time and effort are expended to obtain a workable layout at a minimum cost. The engineer has prided himself on being able to "build for one dollar what any other fool can build for two dollars." The *raison d'être* of the engineer, particularly the consulting engineer who engages in the design and supervision of construction, is his ability to produce a workable design at a minimum cost of construction and operation.

Messrs. Bogert and Greig profess to see in the writer's formulas a "danger of carrying formulary regimentation into the realm of practical engineering," presumably because of the lack of "economy of such formulas in office use." It is not necessary to use the formulas themselves for office use. They merely express the fundamental relations. Any engineer who desires to apply eco-

NOTE.—The paper by Thomas R. Camp, M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Messrs. Clinton L. Bogert, J. M. M. Greig, Charles M. Mower, Jr., and A. C. Michael; and May, 1938, by Messrs. Ellwood H. Aldrich, M. H. Klegerman, and F. Knapp.

[19] Associate Prof., San. Eng., Dept. of Civ. and San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[19a] Received by the Secretary November 12, 1938.

nomic principles to the design of pipe networks may simplify the use of the formulas, or other similar ones, by preparing charts or slide-rules. This is what has long since been done for many of the complicated pipe-friction formulas, as has been so well pointed out by Mr. Klegerman. The time required for the use of the formulas for economic pipe size will be quite short as compared to the time that will be required to solve for the flow distribution after the selection of each trial set of economic pipe sizes. The writer is not attempting to submit a method of design that is easier than methods now in use. He is submitting fundamentals underlying a procedure which heretofore has not been used at all for complex networks.

The client can well afford to pay the cost of many times the office work usually required for the design of distribution system reinforcements if this additional work results in substantial savings in over-all costs. That substantial savings are possible cannot be doubted, because the sums involved in distribution system construction are tremendous. C. Maxwell Stanley,[20] Assoc. M. Am. Soc. C. E., has shown that the costs of a number of distribution systems in Iowa represent between 41% and 85% of the total values of the water-works. According to George A. Sampson[21] the distributing pipes represent about 90% of the value of small New England water-works; and according to W. W. Brush,[22] M. Am. Soc. C. E., about 50% of New York City's water-works investment of a half billion dollars is in the distributing system. Since the cost of water-works is from about $24 to about $70 per person served, the enormous investments involved in distribution pipes are quite apparent.

Messrs. Bogert and Greig, and Mr. Michael, criticize the writer's formulas because the coefficients, costs, and demand rates are not constant and are difficult to estimate. This is true, of course, for any method of design which involves predictions of future conditions. Nevertheless, it is necessary to make such predictions in order to proceed. Fortunately, the effect of errors in these estimates is not so great upon the economics as it is upon the workability of the design. The cost of a pipe line and pumping, plotted against the diameter, results in a **U**-shaped curve which is quite flat in the immediate vicinity of the most economic diameter. The cost, therefore, is not affected appreciably by the selection of the commercial diameter nearest the economic size. Similarly, errors in the estimation of any of the coefficients or other factors to be introduced into the formulas do not result in errors of the same magnitude in the diameter or cost.

An error of only 5% in the economic diameter of pipe requires, for its production, much larger errors in each of the factors in the formulas. The estimated discharge must be in error by at least 11%; the friction factor, C, by at least 18%; the pumping cost by at least 35%; the rate of interest plus depreciation by at least 35%; and the cost of laying, the cost of cast iron, or the value of Factor F, must be in error by more than 35 per cent. Hence, precision in the use of the formulas is not warranted.

[20] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1609.

[21] Paper presented at the September, 1938, meeting of the New England Water Works Assoc. (Not yet published.)

[22] *Loc cit.*

For the same reason it is of little consequence whether total pumping cost, or only power cost, is used; and the error involved in the assumption that total pumping cost is directly proportional to the head and to the discharge is of little moment. These points have been recognized by Mr. Mower and by Mr. Aldrich.

Messrs. Bogert and Greig, and Mr. Aldrich, are concerned regarding the assumption that the cost of elevated storage is unaffected by changes in pipe size. As stated by Mr. Aldrich, "one of the principal purposes of storage is to permit a decrease in pipe capacity." This is correct for the pipes from the supply to storage, but if the storage is more than adequate for hourly fluctuations in demand, the quantity of storage is not related to pipe sizes. Hence, the cost of storage is not related to pipe size, except as the cost is influenced by the elevation of the stored water. The foregoing considerations are based upon the assumption that the locations of the reservoirs are fixed by the topography.

If the locations of the distributing reservoirs are not fixed by the topography, a much more complicated economic problem arises. In flat terrain where elevated storage must be provided in elevated tanks, the choice of the number of tanks and their locations is unlimited. In such a case economic pipe sizes are certainly affected by the location of the storage. The economic considerations leading to the location of distributing reservoirs are outside the scope of the paper. It is manifestly impossible, in one paper, to consider all the possible cases and conditions that might arise in the design of such complex works as distribution systems.

Mr. Michael suggests that "it is conceivable that a pipe having an economic diameter determined by the method described by the author, would not have sufficient capacity to supply the maximum hourly demand." Since the capacity of a pipe depends upon the head provided for flow and the writer's method fixes the head by fixing the elevation of the storage, this difficulty will not arise. It will be found occasionally, however, for cases with widely fluctuating draft, that pressures in the mains will be so great that the computations must be revised using heavier pipe than was originally contemplated. In some cases it will also be found that pumping heads are so great that it will be necessary to revise the computations with the additional pumping equipment costs taken into account.

Mr. Aldrich states that the so-called unpredictable costs, such as rock excavation, wet work, removal and relaying of pavement, and avoidance of obstructions are not independent of pipe size as claimed by the writer, but vary with the size in about the same ratio as the cost of laying. He is quite correct in the case of pavement, but the cost of pavement removal and replacement is predictable and may well be included in the equations of cost. The cost of wet work (sheeting, bracing, and pumping) and avoidance of obstructions is, in most cases, influenced very little by differences in pipe sizes. The cost of rock excavation is influenced by pipe size, but it is not directly proportional to pipe size. According to Mr. Sampson[21] the rock excavation for a number of New England systems averaged 150 cu yd per mile. The cost of this quantity of rock is less than 8% of the total cost of the mains. Where rock excavation is

known to be required in considerable quantities, it can be included in the equations of cost.

The effect of compound interest upon the economic pipe size is of little consequence. It has been accounted for in the writer's equations in the form of depreciation. The factor, r, is the rate of interest plus depreciation. Large errors in the assumed value of r are required to produce appreciable errors in the economic pipe size.

Mr. Klegerman has performed a service in emphasizing the possibilities of using the methods presented in this paper for valuation work. It is probable that many difficulties would arise in securing the acceptance of the procedure outlined by Mr. Klegerman in Court cases, but as the method of hydraulic and economic analysis of distribution networks is perfected and simplified, it is certain to find a place in valuation work.

Mr. Knapp, quite properly, questions the validity of the practice of "skeletonizing" the distribution system when there is little difference in size between the trunk lines and the branch pipes. This criticism applies to the hydraulic analysis only, since the size of the branch pipes is commonly fixed at a minimum consistent with good fire protection (6 in. in the United States) without regard to economic diameter. In spite of the fact that the writer has quite naively described the "skeletonizing" process as a part of the method of attack in design, and despite the fact that much has been written by others relative to "skeletonizing" and many engineers have resorted to this practice, it is certainly not permissible to "skeletonize" any system without some knowledge of the magnitude of the errors that will thus be introduced into the hydraulic computations. So far as the writer is aware, no practical method has yet been presented for determining the magnitude of the errors due to the neglect of the smaller pipes in the hydraulic computations of a grid system. This problem appears to be one which has not yet been properly examined.

Rules (1) and (2) for estimating the take-off power loss are somewhat involved and have doubtless not been stated as clearly as is desirable.[22a] Two rules are required since the power is a function of both head and discharge and estimates of each must be made. The excess head at Point b is represented by the height of the heavy line, h_b, on Fig. 2(b). This head is all chargeable to Pipe 1, for if this pipe were made large enough, this head loss could be avoided. No more than this amount of head loss could be avoided by such procedure, however, without infringing upon the required residual head at Point b. The heads chargeable to the other pipes are shown on Fig. 2(b) as h_2, h_d, h_4, and h_5 at the up-stream ends of the pipes. The power losses corresponding to each of these heads are estimated from the average rates of discharge which must be pumped against these heads and which are taken out of the system at points up stream from the corresponding pipes.

[22a] Corrections for *Transactions:* The captions for Figs. 2 and 4 are correct, but the drawings should be interchanged; and, in the caption for Fig. 3, change " For Case III " to read " For Case II."

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DISCUSSIONS

EARTHQUAKE STRESSES IN AN ARCH DAM

Discussion

BY I. M. NELIDOV AND H. E. VON BERGEN, ASSOCIATE MEMBERS, AM. SOC. C. E.

I. M. NELIDOV[18] AND H. E. VON BERGEN,[19] ASSOCIATE MEMBERS, AM. SOC. C. E. (by letter).[19a]—The several discussions have been gratifying in that they have expanded and clarified the work of the writers. Since many forces and actions come into play, and act upon a dam and its foundation and abutments during an earthquake, it is possible that the title of the paper has been somewhat misleading as to scope. It was the intention, however, to consider the fundamental force of inertia which is created whenever an earthquake tends to accelerate a structure in any direction. The writers consider only the single arch ring as a separate entity, being either a part of a single arch dam or a multiple-arch dam, realizing at the same time that yielding of abutments, stability of buttresses, differential movements of the abutments, hydrodynamic pressures, etc., must be considered to arrive at a complete solution of the problem.

Although the hydrodynamic pressures on dams as proposed by Dean Westergaard[20] are not strictly applicable to arch dams, as stated by Mr. Floris, it may be possible to apply these forces to the face of an arch dam, by some rational method.

Mr. Pearce has directed attention to the unstable condition of a dam in the event of a down-stream acceleration with the reservoir empty. This is especially true for a single arch of the variable-radius design since a dam of this type often is provided with up-stream buttresses to prevent toppling up stream when the reservoir is empty even under static conditions.

NOTE.—The paper by Ivan M. Nelidov and Harold E. von Bergen, Assoc. Members, Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Messrs. A. Floris, and Cecil E. Pearce; and May, 1938, by Messrs. F. W. Hanna, A. W. Fischer, and Ray L. Allin.

[18] Senior Engr. of Hydr. Structure Design, State Dept. of Public Works, Sacramento, Calif.

[19] Asst. Hydr. Engr., Div. of Water Resources, State Dept. of Water Rights, Sacramento, Calif.

[19a] Received by the Secretary October 31, 1938.

[20] "Water Pressure on Dams During Earthquakes," by H. M. Westergaard, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 418.

The writers wish to express their appreciation to Mr. Hanna for clarifying the action of the elementary inertia force and its various components in the third and fourth paragraph of his discussion and as shown in Figs. 4 and 5. Mr. Hanna has also described the use of the equations of moment, thrust, and shear for thick arches as well as thin arches. The error involved in substituting r' for r may be calculated from Equation (34). Another error involved is the appreciable displacement of the neutral axis as the $\frac{t}{r}$-ratio increases above 0.3.[21]

Mr. Fischer discussed the movement of an abutment which was somewhat beyond the scope of the paper. The yielding or movements of the abutments of an arch is a study in itself, not to mention the differential or relative movements of the two abutments during an earthquake.

Mr. Allin mentions the possibilities in the event of an empty reservoir during a lateral earth shock, when the abutments are unable to take tension. He rightly states the effect on the resisting abutment and the crown section and the resulting stresses. These conditions would have to be considered in the design and either special anchorage or increased strength provided.

In conclusion, the writers wish to thank those who have contributed discussions to the paper and thereby have enhanced its value. There is still much to be investigated in regard to forces and other phenomena produced by earthquakes which affect the stability of an arch dam.

[21] "Stresses in Thick Arches of Dams," by B. F. Jakobsen, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 483.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

GRAPHICAL REPRESENTATION OF THE MECHANICAL ANALYSES OF SOILS

Discussion

By Frank B. Campbell, Assoc. M. Am. Soc. C. E.

Frank B. Campbell,[36] Assoc. M. Am. Soc. C. E. (by letter).[36a]—It is felt that the paper has been worth while if only for the interesting and valuable discussion that it has provoked. There appears to be a general desire that something should be done to bring order out of the multitude of systems in current use for the recording and the nomenclature of particle-size distribution in soils. The paper was not presented (as some one has suggested) to add another method to an already "muddled" field, but rather as a logical compromise of the two principal existing systems.

The paper presented four proposals or conceptions regarding the mechanical analyses of soils:

(1) A proposal for the standardization of the form of the mechanical analysis graph;

(2) A proposal for the standardization of the nomenclature of soil fractions based upon a compromise between the American, or Whitney, scheme and the German, or Atterberg, scheme;

(3) A method for expressing approximate distributions of particle sizes and the mean size for use on maps, cross-sections, or tabulations (this proposal is offered as a tool to be used only when found convenient for the sorting of a large number of mechanical analyses); and,

(4) A suggestion as to the interpretation of the mechanical analysis graph in terms of the history of the soil.

Standardization of the Graph.—It is indeed fortunate that the vertical ordinate of percentage by weight smaller than a given particle size, plotted with

Note.—The paper by Frank B. Campbell, Assoc. M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1938, by Messrs. Donald M. Burmister and A. J. Weinig, Jr.; March, 1938, by Messrs. Carl H. Kadie. Jr., Carlton S. Proctor, T. T. Knappen, Jacob Feld, and Howard F. Peckworth; May, 1938, by Messrs. Joel D. Justin, L. B. Olmstead, T. A. Middlebrooks, and Frank E. Fahlquist and Waldo I. Kenerson; June, 1938, by Messrs. E. W. Lane, F. J. Sanger, and F. Knapp; and September, 1938, by Charles H. Lee, M. Am. Soc. C. E.

[36] Hydr. Engr., War Dept., U. S. Engr. Office, Dennison, Tex.

[36a] Received by the Secretary October 24, 1938.

increasing magnitude upward, has found fairly common acceptance. This leaves, then, only the sense, vectorially speaking, of the horizontal ordinate or abscissa to be considered.

In closing, it should be emphasized that some engineers and scientists plot their abscissas with increasing magnitude to the right, whereas others plot them with increasing magnitude to the left. As Mr. Justin has so aptly expressed it, "This seems like a small matter, but when one becomes familiar with the meaning of curves plotted in a certain manner and another technician appears, who insists on doing it differently, it takes some time before the curves mean very much." Professor Burmister and Messrs. Fahlquist and Kenerson have shown graphs with increasing magnitude toward the left. Messrs. Kadie and Lee as well as the writer, have plotted grain size as abscissas with increasing magnitude toward the right.

For the purpose of this discussion the writer will define a positive sense of the abscissa as increasing magnitude toward the right and a negative sense as increasing magnitude toward the left. There may be a good reason for plotting with a negative sense but, unfortunately, no one has offered such reason. The writer feels that originally some one may have had the idea that the upper limit of particle sizes in soils is definite and that the lower limit is indefinite, due to the advancing technique of modern science, which may permit the measurement of smaller diameters in the future. There is probably a definite lower limit if sedimentation methods are to be used. Mr. R. C. Thoreen[37] has clearly defined the limitations of Stokes' law. It would appear then, that the limitation of practical methods of measuring small sizes would indicate plotting the smallest sizes at the left. Considering the fact that both Europeans and English speaking people read with a positive sense from left to right, it would seem more in agreement with their sense of direction if data were plotted with increasing magnitudes toward the right. Furthermore, even the layman is familiar with the classical "first quadrant" of plane geometry. There would probably be less mental confusion for the technician who plots with a negative sense, to change to a graph using a positive sense, than for the technician who has already employed a positive sense, to reverse his procedure. For this reason the writer is presenting a suggested standard form, shown in Fig. 7. It would be somewhat inconvenient for the engineer or scientist who only occasionally plots grain-sized distribution to use a negative sense, in that the semi-logarithmic paper usually available on the market is constructed with a positive sense. Although there is no zero to the logarithmic scale, the left-hand margin of the graph could be started at 0.002 mm, or 2 microns, for all practical purposes. It is fairly positive that sedimentation methods will not be used to determine diameters smaller than 0.6 micron. The expensive and tedious methods of microscopy may yield determinations smaller than the aforementioned limit, but this remains for scientists of the future to determine.

Messrs. Kadie and Lee have called attention to the fact that Fig. 1 does not have an adequate number of abscissa lines for close plotting. Their contention is true; Fig. 1 was included for demonstration purposes only. The writer is

[37] "Comments on the Hydrometer Method of Mechanical Analysis," by R. C. Thoreen, *Public Roads*, Vol. 14, pp. 93–105.

accustomed to using graph paper similar to that shown by Mr. Kadie for purposes of making an accurate record of mechanical analysis data. The method of using heavy lines to correspond to sieve openings of the Tyler series has been found to be very convenient for the purpose of plotting the sieve-analysis data.

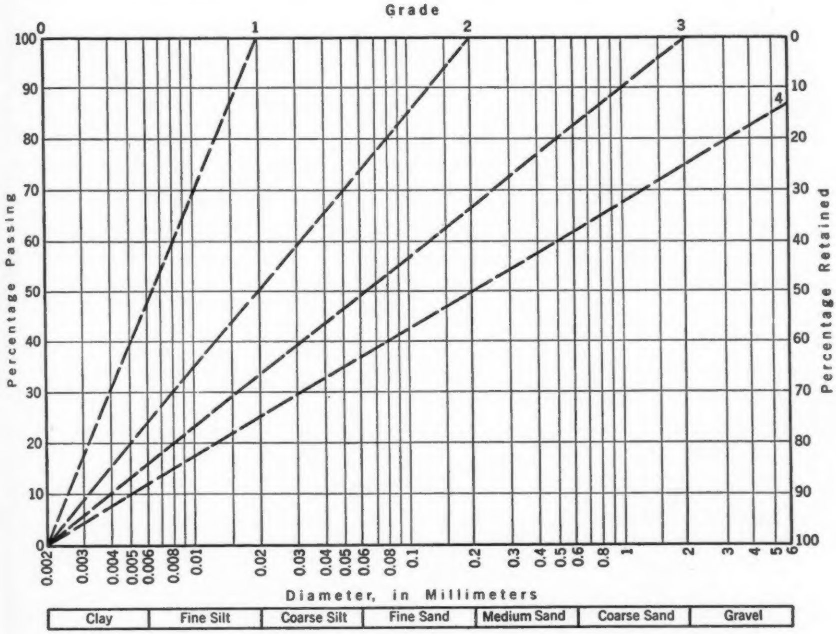

FIG. 7

Grade Line-Mean Diameter.—During the winter of 1934–1935 the Platte Valley Public Power and Irrigation District employed a number of inspectors in its Soils Laboratory under the direction of the writer. With adequate assistance it was possible to make about 700 mechanical analyses on borrow-pit material as well as other tests, such as Proctor compaction and percolation tests, before the ground thawed and construction was begun in the spring. At that time, the writer devised a classification which can be called an A B C classification for the wind-blown or aeolian material in Western Nebraska. This classification was used merely as a tool for plotting the results of mechanical analyses on a plan of the dam. Fig. 8 was a part of one of four borrow-pit maps used to guide the engineers and inspectors in directing the contractor's excavating equipment. The following tabulation is a grade-line-mean-diameter designation of the A B C classification:

Local classification	Proposed Designation: Grade line	Mean diameter
A	0.60	0.041
B	0.85	0.080
C	1.60	0.145

By studying Fig. 8, it may be seen that the A B C classification was very convenient for designating fine, medium, and coarse material. One might also choose to designate the three classes by the initial letters, F M C. The designations, A B C, may be confused with soil horizon designations by soil surveyors.

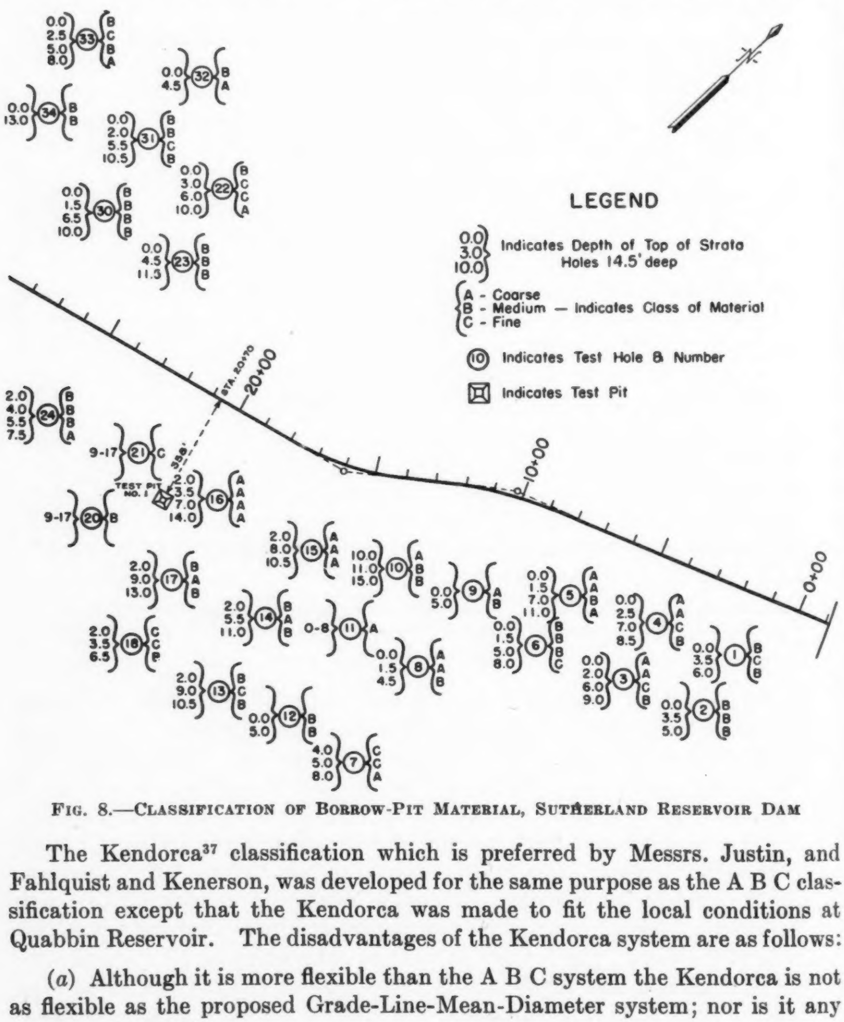

FIG. 8.—CLASSIFICATION OF BORROW-PIT MATERIAL, SUTHERLAND RESERVOIR DAM

The Kendorca[37] classification which is preferred by Messrs. Justin, and Fahlquist and Kenerson, was developed for the same purpose as the A B C classification except that the Kendorca was made to fit the local conditions at Quabbin Reservoir. The disadvantages of the Kendorca system are as follows:

(*a*) Although it is more flexible than the A B C system the Kendorca is not as flexible as the proposed Grade-Line-Mean-Diameter system; nor is it any more precise; and,

(*b*) The system is incomplete in that odd numbers do not indicate the existence of any material between the 100% and 50% lines.

Regardless of the two foregoing disadvantages the writer can readily understand that the Kendorca classification may have been very valuable as a tool for the materials problem which the originators had to solve. However, it is believed that if one becomes accustomed to working and speaking with the

terms of the 20–80 Grade-Line-Mean-Diameter system it will be found more flexible than establishing some fixed line on a mechanical analysis graph. The modifications proposed by Professor Lane are well worth considering. The writer believes that the use of 10% and 90% intersection points for establishing a grade line would be workable for soils that have the curve symmetrical about the 50% axis. The disadvantage of using 10% and 90% would be the large deviation introduced at 50% mean diameter when the distribution curve has either exaggerated convexity or concavity. Professor Lane's suggestion of using 6/6 of the ratio of the interval rather than 10/6 would be merely another mathematical designation of the grain-sized distribution. It would have the advantage, as he suggests, of enabling the computer to make a hurried mental calculation of the grade instead of resorting to graphical methods. Professor Weinig's method of adding symbols to the diameter grade designation to indicate convexity, concavity, *S*-type, and linear curve, is certainly an excellent suggestion. It would probably add much to the value of the data on a plan or cross-section if the soil presented these four different types of particle-size distribution.

Nomenclature of Soil Fraction.—There appears to be an attitude, as expressed by Messrs. Knapp, Knappen, and Sanger, that the generic names of soil fractions on separates should be discarded entirely. However strongly one may feel toward discarding the English words—colloids, clay, silt, sand, and gravel—they have grown to have a definite meaning of magnitude through the widespread use of mechanical analyses. The general difficulty appears to arise in attempting to reconcile the German *schlamm*, *schluff*, *mo*, and *sande*, to the conception of English-speaking people. If it should be agreed by an international commission on soil nomenclature to establish names for soil separates, with due consideration to the generic meaning of both German and English terms, it may be desirable to compromise by designating the fractions by the letters of the Roman alphabet which is acceptable in both languages. For instance, one could call colloids the *a* fraction, clay the *b* fraction, silt the *c* fraction, sand the *d* fraction, and gravel the *e* fraction. There would then remain only the problem of standardizing the size limits. Another alternative would be to use the Greek alphabet in the manner that radio-active rays are designated. On the other hand, the American and English lexicographers would probably appreciate precise definitions of generic words which have heretofore been used very loosely by scientists, engineers, and laymen alike.

Mr. Middlebrooks raises a question as to the reason for changing from the Bureau of Chemistry and Soils classification. The writer believes that the reason for changing is that a large group of engineers and scientists have already begun to swing toward the more logical scheme of Atterberg by setting the division lines at recurring logarithmic cycles in multiples of 2. As stated by Professor Lane, Atterberg's[38] original classification called for the upper limit of clay to be drawn at 2 microns. The same limits established by Atterberg have been followed by the International Classification.

[38] "Die rationelle Klassifikation der Sande und Kiese," von A. Atterberg, *Chemische Zeitung*, Jahrgang 29, 1905, pp. 195–198.

Few engineers and scientists in America have become accustomed to the International, or the Atterberg, designation until recently. In the meantime the Americans have used the U. S. Bureau of Chemistry and Soils Classification for many years. The writer is indebted to Mr. Olmstead[39] for a brief history of the Bureau of Chemistry and Soils Classification:

"The earliest publication I have seen that gives the Bureau grain size classification is the report of Professor Milton Whitney in the Fourth Annual Report of the Maryland State Agricultural Experiment Station, 1891, pages 249–296. *Bulletin 4* of the Division of Agricultural Soils, published in 1896, states that it is the practise of the Division to separate mineral soils into eight grades, according to diameter of grains, and gives the same limits as does the Maryland station report. No author's name appears on this *bulletin*. Osborne employs some of the same size limits in his mechanical analyses of soils published in the Connecticut Agricultural Experiment Annual Reports for 1886–7. I have seen no discussion of the origin of the Bureau classification but I believe it was devised by Professor Whitney some time before he came to the Department of Agriculture as the first chief of the Bureau of Soils."

The Bureau of Soils classification, therefore, is distinctly American, having its origin sometime during the latter part of the Nineteenth Century. Many English-speaking scientists have become accustomed to it. On the other hand, there is the more rational classification as far as limits or divisions of the soil fractions, which apparently originates with Atterberg.

As mentioned in Mr. Olmstead's discussion, the engineers would probably prefer to have more divisions in the coarse range whereas the agronomist would prefer more divisions in the fine range. In the proposed classification, an attempt has been made to establish limits for fine, medium, and coarse sand, which roughly approximates that of the Whitney scheme except log 2 – log 6 has been used for the limits. More divisions than the old Whitney system have been made available in the range of clay and silt. For these reasons it should be usable by both engineers and agronomists.

A comparison of the advantages and disadvantages of the several systems can be outlined briefly as follows:

I.—Atterberg or International Classification:

(*A*) Advantages.—

(1) The class limits are set at 2, 20, 200, and 2 000 microns making a good arrangement for semi-logarithmic plotting.

(*B*) Disadvantages.—

(1) There is some difficulty of applying the German terminology to the American concepts of size classification.

(2) There are too few divisions in the coarser range.

(3) The upper limits of clay at 2 microns is too high for efficient laboratory determination where a large number of samples are involved. The laboratory time required for the Bouyoucos hydrometer is approximately 6 hr.

[39] Personal letter to the writer from L. B. Olmstead, Soil Physicist, U. S. Bureau of Soils, Washington, D. C., dated October 12, 1938.

II.—Whitney or Bureau of Chemistry and Soils Classification:

(*A*) Advantages.—

(1) There is a widespread familiarity with this system among American professional men and perhaps also laymen.

(2) The numerous divisions in the larger materials make it convenient for macroscopic examination of soils in the field by engineers and soil surveyors.

(3) It establishes a very practical upper limit for clay whether the pipette or hydrometer method is used.

(*B*) Disadvantages.—

(1) The size limits of the class do not conform to evenly divided logarithmic spacings. Whitney's proposal was probably originated before the common use of semi-logarithmic plotting.

(2) Too few divisions in the smaller size are included. This is a disadvantage to the agronomist principally, although others may soon be interested, with the advance of the study of soil mechanics.

III.—Massachusetts Institute of Technology Classification:

(*A*) Advantages.—

(1) Sand is divided into three commonly accepted classifications; namely, fine, medium, and coarse.

(2) Sand classes roughly approximate those of the Bureau of Chemistry and Soils.

(*B*) Disadvantages.—

(1) There are three classes, each, of silt and clay. The three classes of clay have as yet proved to be of little use to the engineer.

(2) This has the same disadvantage as the International in that the upper limit of clay is depressed to 2 microns instead of at the proximity of 5 microns according to the Whitney system.

IV.—Proposed or Log 2 − Log 6 Classification:

(*A*) Advantages.—

(1) This is a classification to agree roughly with the Whitney system which has grown to be the American conception of size classification.

(2) It is a division of soil fraction based upon log 2 − log 6 which is patterned after the Atterberg system.

(3) An upper limit of clay is set at 6 microns instead of 2 or 5 microns. Such a definition allows clay to be determined in less than 1 hr with a Bouyoucos hydrometer as compared with more than 6 hr for the Atterberg definition. For the investigation of any particular borrow-pit, where the number of samples to be examined is known, this advantage

can be evaluated in terms of dollars and cents. With the more cumbersome pipette method, a shorter distance of settlement can be used, but accuracy is sacrificed. The Bouyoucos hydrometer method is usually conceded to be a more efficient method, although the pipette is desired by some technicians.

(*B*) Disadvantages.—

(1) The writer feels that a distinct difficulty exists in that a compromise is suggested between the two schools of thought; namely, those who are experienced with the Whitney system and those who have become accustomed in recent years to the Atterberg system with its modifications.

History of Soils.—Professor Burmister and Mr. Sanger obviously concur with the writer that the mechanical analysis is an indication of the history of the soil.

Mr. Lee dissents with a statement that the premise given by the writer may not be true in the case of resistant quartz or silica. Although quartz may be more resistant to weather than other minerals, it has been observed that even such hard crystals are vulnerable to breakage by freezing and thawing of interstitial water. Even diurnal variation of temperature in the warmer latitudes may cause the disintegration of quartz. This is widely evidenced in the Piedmont region of Southeastern United States. However, Mr. Lee's argument should be considered because the writer's suggestion as to maturity and immaturity of soils is probably more applicable to the podzols of the northern latitudes.

Messrs. Feld and Sanger have mentioned the shape of particles. This fact is important in that, the determination of mechanical analyses by a combination of sieving and sedimentation methods does not take the shape into account. Some one has well defined a diameter determined by sedimentation methods as the "equivalent diameter of a perfect sphere falling in a viscous fluid." As a general rule the presence of angular and plate-shaped particles increase the stability of embankments, although more effort may be required for the mechanical compaction of such soils. The angular particles may require more fines to fill the voids and produce imperviousness. It has been the writer's practise to carry a pocket microscope giving magnification to approximately 60 diameters, when working in the field. Thus, the shape of particles of approximately 20 microns may be inspected visibly. This diameter is the lower limit of coarse silt on the proposed classification.

General Consideration.—In closing, the writer suggests that individuals, interested societies, governmental agencies, and educational institutions should work together in drafting a standard form of mechanical analysis graph, which, in effect, is a language used by many professional men. Consider the confusion if one were to attempt to write the words "mechanical analyses" backward as follows: "sesylana lacinahcem."

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DISCUSSIONS

NATURAL PERIODS OF UNIFORM CANTILEVER BEAMS

Discussion

By Merit P. White, Jun. Am. Soc. C. E.

Merit P. White,[16] Jun. Am. Soc. C. E. (by letter).[16a]—Since this paper is intended primarily for the designer of earthquake-resistant buildings, a question arises as to the errors involved in treating an ordinary building, even one of constant cross-section, as a uniform cantilever beam with definite shear and bending flexibility. On the other hand, even a very simple building is a tremendously complicated elastic-dynamic system. The interaction of frame, walls, and partitions, the variations in stiffness of these elements due to doors and windows, the concentrations of mass at the floor levels—these and similar factors make impossible any mathematically exact analysis. Obviously, in order to secure any information at all on the dynamical characteristics of a building many simplifying assumptions are necessary.

In his "Synopsis" the author states that close agreement with actuality is not to be expected if the theory is applied to buildings. The writer agrees with this statement to the following extent: Certainly, any one possessing only a complete set of plans for a given building, including data on the deformation characteristics of the site, and with no empirical information obtained by observations on similar buildings on similar sites, would be fortunate indeed to be able to predict within 20% the fundamental period of the building in question. On the other hand, possession of the empirical information just mentioned would permit a much closer prediction of the building period. Furthermore, there are certain other, very important quantities, such as the shapes of the several modes of vibration, the relation between the fundamental period and the periods of higher modes, and the change in building period due to foundation yielding, all of which, the writer believes, can be determined with considerable accuracy from the theory presented by Professor Jacobsen.

Note.—The paper by Lydik S. Jacobsen, Esq., was published in March, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by K. Bert Hirashima, Esq.

[16] California Inst. of Technology, Pasadena, Calif.

[16a] Received by the Secretary October 29, 1938.

Although it is quite true that an ordinary building is so complicated that any attempt to make a mathematically exact analysis in an endeavor to test the validity of the author's simplifying assumptions would be an endless task, nevertheless it may be possible to check these assumptions by making others, quite different, and performing the calculation which they make possible.

The author simplified his problem by replacing discontinuous distributions of weight and stiffness by continuous functions. Actually, these characteristics were considered constant. Another possible simplification would be the following: Consider all weights to be concentrated at the floors, and replace the stiffness of frame, walls, and partitions by fixed-ended columns. It must not be thought that the fixed-ended columns in the proposed model are identical with the columns of the real building. Actually, they are much stiffer because the greatest part of the lateral stiffness of an ordinary building is due to its walls and partitions. This can be shown by comparing measured building periods with those computed, neglecting the stiffening of the walls. These hypothetical columns are purely schematic and are intended to duplicate the stiffness, and not the structure, of a building.

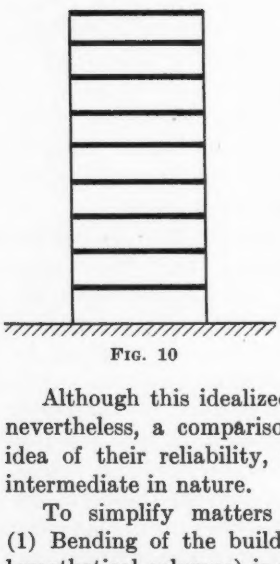

Fig. 10

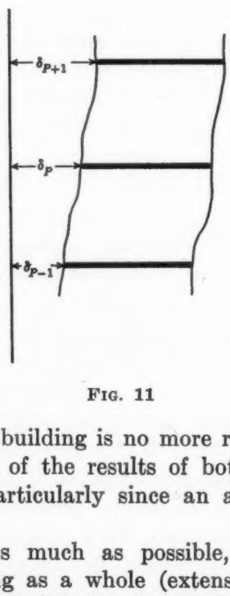

Fig. 11

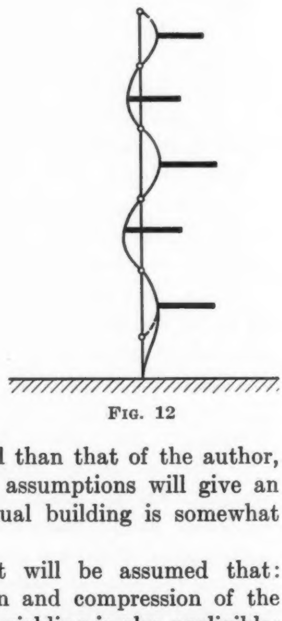

Fig. 12

Although this idealized building is no more real than that of the author, nevertheless, a comparison of the results of both assumptions will give an idea of their reliability, particularly since an actual building is somewhat intermediate in nature.

To simplify matters as much as possible, it will be assumed that: (1) Bending of the building as a whole (extension and compression of the hypothetical columns) is negligible; (2) foundation yielding is also negligible; (3) the weights of all floors are equal; and (4) all stories are equally stiff. Although these limitations detract much from the reality of the model, if the agreement in this very particular case is satisfactory, it may safely be assumed that the author's assumptions are generally permissible.

The following problem is to be solved: To find expressions for the frequencies of the several modes of vibration of the structure shown in Fig. 10, consisting

of a given number (N) of floors, each of Mass m, supported by columns giving a stiffness, K, to each story (K is the force, in pounds, which will produce unit relative deflection between two adjacent floors). Since N quantities (the deflections of the several masses, for example) are necessary to describe the configuration of the system completely at any instant, the system is said to have N degrees of freedom. Thus, there will be N different modes and frequencies of vibration which can be calculated by solving a system of N simultaneous linear equations.

Except for the boundary condition at the roof, this problem is identical with that of finding the modes and frequencies of vibration of a series of identical, equally spaced beads on a stretched string, first solved by Lagrange.[17] In fact the solutions to the present problem are identical with half the solutions of Lagrange, those possessing an even number of nodes or an odd number of half waves. This follows from the fact that the boundary condition at the top of a vibrating building—namely, that at this point the shear is zero—is satisfied at the center of a vibrating string of beads provided this point is not a node. The computation is quite simple.

Assume that the system shown in Fig. 10 is vibrating in any one of its normal modes (fundamental, second harmonic, etc.). In such a vibration each point moves with simple harmonic motion, of frequency, f_n, this being the natural frequency of the mode in question (the nth mode). Furthermore, in any normal vibration all points move in phase; that is, they pass through their extreme positions simultaneously. If δ_p is the amplitude of motion of the pth mass, then its displacement at any instant is,

$$\Delta = \delta_p \sin (\lambda_n t - \xi_n) \qquad (55)$$

in which ξ_n is the phase angle for the nth mode of vibration and is the same for all points of the structure; and, λ_n is equal to $2 \pi f_n$. By differentiating the expression for displacement, it can be shown that the acceleration of the pth mass is: $- \lambda_n^2 \delta_p \sin (\lambda_n t - \xi_n)$. This acceleration is always equal to λ_n^2 times the displacement and is in the opposite direction.

The present problem is to determine f_n—in other words, to calculate the frequency of the nth mode of vibration. Fig. 11 shows the masses at the points, $p - 1$, p, and $p + 1$, in their extreme positions. Consider the equilibrium of the pth mass on which the following forces act: (1) A D'Alembert force equal to mass times acceleration; (2) a force due to bending of the columns above; and (3) the same due to the columns below.

For the pth mass,

$$m \lambda_n^2 \delta_p + K (\delta_{p+1} - 2 \delta_p + \delta_{p-1}) = 0 \qquad (56a)$$

For the mass at the top (the Nth),

$$m \lambda_n^2 \delta_N + K (- \delta_N + \delta_{N-1}) = 0 \qquad (56b)$$

and, for the lowest mass (the first),

$$m \lambda_n^2 \delta_1 + K (\delta_2 - 2 \delta_1) = 0 \qquad (56c)$$

[17] "Mechanique Analytique," by Joseph L. Lagrange, Tome I, p. 390.

Making $2 - \frac{m}{K}\lambda_n^2 = C$, the complete set of N-equations can be written:

$$\left.\begin{array}{l} C\,\delta_1 - \delta_2 = 0 \\ -\delta_1 + C\,\delta_2 - \delta_3 = 0 \\ -\delta_2 + C\,\delta_3 - \delta_4 = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ -\delta_{N-2} + C\,\delta_{N-1} - \delta_N = 0 \\ -\delta_{N-1} + (C-1)\,\delta_N = 0 \end{array}\right\}\ldots(57)$$

The homogeneity of Equations (57) arises from the fact that all terms contain δ. Thus, the shape, but not the absolute dimensions, of the mode of vibration can be found. One of the δ-ratios will remain arbitrary. The remaining δ-ratios and C are the N unknowns to be determined.

Several methods exist for finding the natural frequencies of the given system from Equations (57). The usual procedure in such a case is based on the fact that the determinant of the coefficients of the unknown amplitudes must be zero in order that the δ-amplitudes may be different from zero. Since the coefficients of the δ-ratios involve only C and unity, it is clear that a single algebraic equation (identical with the determinant of the coefficients) can be obtained. This will be an Nth degree equation in C, the N roots (or solutions) of which will correspond to the N frequencies of the system. By making certain trigonometric substitutions, expressions for the frequencies in the general case (that is, in terms of N) can be found.

A somewhat simpler method, described by Professor A. G. Webster,[18] is the following: The quantity, $C\left(= 2 - \frac{m}{K}\lambda_n^2\right)$, is always less than $+\,2$. This is obvious since m, K and λ_n^2 are all positive. The least value of C occurs with maximum λ_n ($= \lambda_N$). This corresponds to a mode (deflection curve) in which alternate masses are deflected in opposite directions as shown in Fig. 12. The frequency in this case is slightly less than it would be if all columns were pinned at their centers (including a half-column added at the top) as indicated in Fig. 12. Due to the pins, which allow rotation but not displacement of the center point of each column, certain constraints would be imposed on the system, thus raising its frequency. In the case of the pinned columns it is seen that the system is equivalent to N one-degree-of-freedom systems, each of frequency, $\lambda' = 2\sqrt{\frac{K}{m}}$. Since $\lambda_N < \lambda'$, it follows that $-\,2 < C < +\,2$.

On examining Equations (57) it is seen that each equation except the last resembles the trigonometric equation relating the sines of three consecutive multiples of an angle, θ:

$$-\sin(r-1)\,\theta + 2\cos\theta\sin r\,\theta - \sin(r+1)\,\theta = 0 \ldots\ldots (58)$$

Resemblance becomes identity if: $\delta_r = A\sin r\,\theta$; and $C = 2\cos\theta$. This is permissible since $-\,2 < C < +\,2$. All equations except the Nth are satisfied

[18] "Dynamics," by A. G. Webster, Second Edition, p. 167.

by any value of θ. To satisfy the Nth equation, however, it is clear that only certain values of θ are possible. Rewriting the last of Equations (57):

$$A\left[-\sin (N-1)\theta + 2\cos\theta \sin N\theta - \sin N\theta\right] = 0 \qquad (59)$$

since $-\sin (N-1)\theta + 2\cos\theta\sin N\theta - \sin (N+1)\theta = 0$,

$$\sin N\theta - \sin (N+1)\theta = 0 \qquad (60)$$

is the equation that determines possible values of θ. Therefore, $(2N+1)\theta = \pi, 3\pi, 5\pi \cdots (2n-1)\pi \cdots (2N-1)\pi$; and,

$$\theta = \frac{2n-1}{2N+1}\times\pi; \qquad n = 1, 2, 3 \cdots N \qquad (61)$$

Since $C = 2 - \frac{m}{K}\lambda_n^2 = 2\cos\theta = 2\cos\frac{2n-1}{2N+1}\pi$,

$$\lambda_n^2\frac{m}{K} = 2\left(1-\cos\left(\frac{2n-1}{2N+1}\right)\frac{\pi}{2}\right) = 4\sin^2\left(\frac{2n-1}{2N+1}\right)\frac{\pi}{2} \qquad (62a)$$

and,

$$\lambda_n = 2\sqrt{\frac{K}{m}}\sin\left(\frac{2n-1}{2N+1}\right)\frac{\pi}{2}\,(n = 1, 2, 3, \cdots N) \qquad (62b)$$

For a system of N-masses there will be N-frequencies and modes of vibration, the former obtained by making $n = 1, 2, 3, \cdots N$, in Equation (62*b*).

If $\frac{n}{N}$ is small, Equation (62*b*) becomes approximately:

$$\lambda_n = \sqrt{\frac{K}{m}}\frac{2n-1}{2N}\pi\,(n = 1, 2 \cdots \ll N) \qquad (63)$$

TABLE 4.—VALUES OF $\frac{T}{\sqrt{y_s}}$ FOR DIFFERENT VALUES OF N (NUMBER OF STORIES) AND n (ORDER OF HARMONIC)

Values of of n	Values of N						
	1	2	3	5	10	20	∞
1	0.320	0.299	0.294	0.290	0.289	0.288	0.288
2	....	0.115	0.105	0.100	0.097	0.096	0.096
3	....	....	0.073	0.063	0.059	0.058	0.058
4	....	....	....	0.049	0.043	0.042	0.041
5	....	....	....	0.043	0.035	0.033	0.032

Equation (63) may be compared with the author's Equations (8). In the present case:

$$y_s = \frac{m\,g}{K}(1+2+3+\cdots N) = \frac{m\,g}{2K}(N^2+N) \qquad (64)$$

in which y_s, as defined by the author, is the static deflection, in inches, due to

a horizontal acceleration equal to g. If $N \ll N^2$ this gives, when substituted in Equation (63):

$$T_n = \frac{0.288}{2\,n - 1}\sqrt{y_s} \qquad (65)$$

which is equivalent to Equation (8). In terms of y_s, Equation (62*b*) becomes,

$$\lambda_n = \frac{2.772}{\sqrt{y_s}}\sqrt{N^2 + N}\sin\left(\frac{2\,n - 1}{2\,N + 1}\right)\frac{\pi}{2} \qquad (66)$$

and,

$$\frac{T_n}{\sqrt{y_s}} = \frac{0.226}{\sqrt{N^2 + N}\sin\frac{2n - 1}{2\,N + 1}\frac{\pi}{2}} \qquad (67)$$

Table 4 gives, for comparison, values of $\frac{T}{\sqrt{y_s}}$ for various values of n (order of harmonic) and N (number of floors in the structure). The last column ($N = \infty$) is from Equation (65) and corresponds to the values obtained by the author. It appears that even for structures of one or two stories the agreement is fairly close, while the lower frequencies in structures of several stories agree almost exactly with those given by the author's equations.

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DISCUSSIONS

A THEORY OF SILT TRANSPORTATION

Discussion

BY MESSRS. O. A. FARIS, J. E. CHRISTIANSEN, SAMUEL SHULITS, AND GERALD LACEY

O. A. FARIS,[30] M. AM. SOC. C. E. (by letter).[30a]—The theory of silt transportation suggested by Mr. Griffith may be applicable under the limitations and conditions which he proscribes; however, it is believed that its application to most of the rivers of Texas will ordinarily lead to serious error. Investigations,[31] under the direction of the writer, include the collection and determination of the silt content of thousands of samples of water from various rivers of Texas, over a 9-yr period. The velocity of the water at the point of sampling was determined in many instances.

In sampling a flood originating in the upper regions of a water-shed, at a station farther down stream and out of the storm area which produced the rise, it was found that the first water reaching the station was comparatively free from suspended material. This was due to the arrival of clear water forced out of pools, along the river, by the heavier silt-laden water of the flood. As the stage built up, the silt percentage increased to a certain point and then decreased although the river stage continued to rise. During the falling stage, the percentage of silt again increased to some extent. This secondary increase in silt percentage is believed to be due to the sliding of recently deposited silt, into the stream, from the sloping banks where trees and brush retarded the velocity to the extent that deposition resulted. In some reaches of the streams, the secondary increase in silt percentage is due largely to the caving of banks undercut by the current. The greater part of the silt load, of the streams under observation, is made up in advance by the process of weathering. After a long dry period the first water that runs off picks up the fine weathered material and carries it into the stream. After the first flushing, run-off from the wet area must depend on erosion for its silt load, which is ordinarily light

NOTE.—The paper by W. M. Griffith, Esq., was published in May, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1938, by Joe W. Johnson, Jun. Am. Soc. C. E.; and October, 1938, by Messrs. George W. Howard, Harry F. Blaney, and E. W. Lane.

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[30a] Received by the Secretary October 25, 1938.

[31] "The Silt Load of Texas Streams," by Orville A. Faris, *Technical Bulletin No. 382*, U. S. Dept. of Agriculture, 1933.

since the part of a large water-shed where excessive erosion occurs is usually small in comparison with the entire area. This accounts for the decrease in silt percentage while the river stage continues to rise.

The character of the part of the water-shed on which a flood originates affects the quantity of silt reaching a down-stream station at equal stages of the river. At Waco, Tex., on the Brazos River, it has been observed that floods from the Bosque River which joins the Brazos about 3 miles up stream from the station, contain a lower percentage of silt, at the same stage, than floods originating on the upper part of the Brazos water-shed.

These investigations led to the conclusion that the total quantity of silt passing a station in these streams, at a given time, is a function of loading rather than velocity and depth of water in the section. It is true that the higher the velocity the greater is the carrying capacity; but, since the capacity load was not even approximately reached and the silt content at a single station varied for equal stages of different floods, the magnitude of the silt charge carried appears to be a function of the loading and not of capacity to carry.

J. E. Christiansen,[32] Assoc. M. Am. Soc. C. E. (by letter).[32a]—Following Equation (1) the author states,

"It is claimed that from Chezy's basic equations of flow for unerodible perimeters, the average velocities in the vertical planes through the points, *A*, *B*, *C*, etc. [Fig. 1], across the section will vary as the square root of the depths at those points; that is, as $d_1^{0.5}$, $D_2^{0.5}$, $d_3^{0.5}$, etc."[3]

This is only a rough approximation at best, and cannot be considered a basic law. Mean velocities in vertical planes of channels with flat bottoms often vary considerably from the center of the channel to sections near the sides. In the Chezy formula,

$$V = C\sqrt{RS} \qquad (16)$$

C is not constant; it is a function of S, R, and Kutter's coefficient of rugosity, n, assuming that Kutter's formula represents the relationship correctly. For a given value of Kutter's n, and between limits of R and S, the Kutter-Chezy formula can be approximated closely by simple exponential formulas; for example, for Kutter's $n = 0.0225$, and for slopes greater than 0.0001, the approximate equations are:

For $0.1 < R < 1.0$,

$$V = 66\,R^{0.81}\,S^{0.51} \qquad (17a)$$

for $1.0 < R < 10$,

$$V = 66\,R^{0.69}\,S^{0.51} \qquad (17b)$$

for $10 < R < 100$,

$$V = 74\,R^{0.58}\,S^{0.49} \qquad (17c)$$

Compare Equations (17) with Manning's formula, which for Kutter's $n = 0.0225$ becomes,

$$V = 66\,R^{0.667}\,S^{0.5} \qquad (18)$$

[32] Asst. Irrig. Engr., Coll. of Agriculture, Univ. of California, Davis, Calif.
[32a] Received by the Secretary November 7, 1938.
[3] *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), pp. 245–246, 279.

The author's theory of silt transportation is based fundamentally on a formula of the Kennedy type, Equation (1), in which he assumes that c is primarily a function of the quantity of silt carried. If his reasoning from Equation (1) is correct, the value of n should exceed the value of the exponent of R in Equations (17) and (18). Most of the formulas of the Kennedy type[6] involve the use of exponents of d or R less than the values of the exponents of R in these equations, except Equation (17c), which apply up to values of $R < 10$.

The writer believes that undue importance has been attached to the Kennedy formula. In 1927, discussing the prior paper cited by the author,[4] Mr. F. M. Woods stated[33] that Mr. Griffith had relied greatly on the Kennedy formula and had also extended its application to conditions of great rivers in unstable regime, involving every kind of irregularity of flow. The formula was stated to have been consistent with hydraulic data observed by Mr. Kennedy on a limited range of artificial earth canals; but, to extend its application as described would be questionable even if it were correct for canals in stable regime. According to Mr. Wood, Mr. Kennedy had enunciated no original theory, having merely deduced, from observed data, a useful empirical formula in support of theories current in India, which had been enunciated thirty years earlier by Mr. Thomas Login, an engineer on the Ganges Canal. The latter, in turn, was merely confirming, from his own experiences in the Ganges Canal, theories suggested by Mr. J. Dupuit in 1848. Mr. Woods then proceeds to warn that the Kennedy formula, based on "observations of certain Punjab irrigation-canals," has marked limitations despite the fact that it had been a useful guide to the design of canals for thirty years. For example, Mr. Kennedy did not measure the water-surface slope of any of his sample channels; nor did he compute the corresponding values of Kutter's coefficient of rugosity. He evaluated the discharges from discharge tables and the mean velocities by dividing this discharge by the area of cross-section. According to Mr. Woods, even Mr. Kennedy's determination of area of waterway was not quite satisfactory, being expressed in terms of bed width and depth, without reference to the inclination of the side slopes.

Since the values of the exponents of d or R in both the silt formulas and general flow-of-water formulas are of approximately the same magnitude, the most logical conclusion would appear to be that the slope, for stable channels, should be primarily a function of the type and quantity of silt to be carried. Equating Kennedy's and Manning's formula:

$$C\,d^{0.64} = \frac{1.486\,R^{0.667}\,S^{0.5}}{n} \qquad (19)$$

from which,

$$S = \frac{C^2\,n^2}{2.21}\left(\frac{d^{0.64}}{R^{0.667}}\right)^2 \qquad (20)$$

[6] "Stable Channels in Erodible Material," by E. W. Lane, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 127.

[4] "A Theory of Silt and Scour," by W. M. Griffith, *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927), p. 253.

[33] *Loc. cit.*, p. 268.

Because of the inexactness of the exponents of d and R the ratio, $\frac{d^{0.64}}{R^{0.667}}$, may be assumed to be constant. For a given value of Kutter's n, then:

$$S = K C^2 \qquad (21)$$

This leads to the conclusion that a channel may be in stable equilibrium at any depth providing the slope is correct, a conclusion very different from that reached by the author.

The theory of silt distribution developed by W. Schmidt[34] and M. P. O'Brien, Assoc. M. Am. Soc. C. E.,[35] has been checked against some of the same data[36] used by the author, with a correlation equally as good. This theory does not account for the concentration of silt in suspension, or that which is moved along the bottom; but it does show that the distribution of silt within the section is a function of the size of the silt particles and the turbulence of the stream. From this theory, it would appear questionable to attempt to divide silt arbitrarily into "colloids" which are "carried in suspension by all eddies" and "loose granular material" that is "dependent on the vertical eddies alone." What may be "colloids" in one case may be "loose granular material" in another case, with a different degree of turbulence.

E. W. Lane,[37] M. Am. Soc. C. E., has called attention to the correlation between the quantity of coarse material in suspension and the discharge of a given stream, and the apparent lack of any correlation between the very fine material and the discharge. He attributes this condition to a balance between the kind of material in the bed of the channel and that in suspension. The percentage of very fine material in suspension, of which there is little in the bed material, is governed primarily by the source of such material. The writer came to essentially this same conclusion when studying silt data from the Colorado River where high concentrations of fine material generally occur during the fall of year when the discharge of the Lower Colorado is fairly low. These high concentrations of fine material coincide, however, with the period of thunder-storms in desert areas, and with the flashy peak discharges of some of the lower tributaries that bring this material into the Colorado River. The concentration of the coarser material coincides fairly well with the discharge, but appears also to depend upon whether the river stage is falling or rising. With a sustained high discharge, the concentration of coarse material generally decreases.

Samuel Shulits,[38] Assoc. M. Am. Soc. C. E. (by letter).[38a]—An empirical approach to the problem of the transportation of solids by streams is presented

[34] "Die Massenaustauch in freien Luft und verwandte Erscheinungen," von W. Schmidt, H. Grand, Hamburg, 1925.

[35] "Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Hydrology Section, 1933, p. 487.

[36] "Distribution of Silt in Open Channels," by J. E. Christiansen, *Transactions*, Am. Geophysical Union, Hydrology Section, 1935, p. 478.

[37] "Stable Channels in Erodible Material," by E. W. Lane, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 193.

[38] Asst. Prof. of Mechanics and Hydraulics, in Chg., Fluids Mechanics Laboratory, Colorado School of Mines, Golden, Colo.

[38a] Received by the Secretary November 4, 1938.

in this excellent paper. The value of the method outlined lies in its simplicity and its seeming success in fitting existing data. The writer will attempt to subject certain parts of it to a critical analysis, in the hope that more light will be thrown on the fundamental bases of Mr. Griffith's formula.

The statement that Equation (1) is the elementary law of silt transportation seems to lack rigorous foundation. This equation can be regarded as scarcely more than an empirical formula derived from measured data of stable channels by the principles of mathematical curve-fitting. Accordingly, it cannot be vouchsafed the position of a fundamental law. By varying the exponent, n, the equation can be made to fit any set of data. Comparative compilations of such equations by E. W. Lane,[39] M. Am. Soc. C. E., have revealed such wide disagreement between existing formulas of this type that their practical reliability is rather uncertain. The thoughts expressed in this paragraph also apply to Equation (2).

When it is remembered that Chezy's velocity formula is based on the arbitrary assumption that the tractive force is porportional to the square of the mean velocity, it would appear that the criterion of $n = 0.5$ for equilibrium (or for channel stability) may not be well founded. Furthermore, since the tractive force is equal to $w\,R\,S$ (in which w is the unit weight of water; R, the hydraulic radius; and S, the energy gradient), considerable error may be incurred by substituting the mean depth, d_m, for the hydraulic radius, especially in small channels.

In natural channels the movement of material on the bed is limited to a strip approximately in the center, no movement occurring near the sides. This has been shown by field measurements made by S. Kurzmann[40] and explained experimentally by A Schoklitsch [41, 42] These results are at variance with Mr. Griffith's deductions that bed movement exists only where his depth criterion based on $n = 0.5$ is exceeded. Actual measurements to substantiate the hypothesis would be extremely valuable.

It is interesting to consider the significance of the stated possibility of mathematical calculation of f, the ratio of the cross-sectional mean velocity to the mean velocity in a particular vertical. This connotes that the mean velocity in any vertical of a cross-section can be predicted accurately.

Tables 1 to 4 merely show that Equation (13) fits the particular data well. One wonders if measurements in a tidal channel with its complex flow conditions should be used as the basis of a general formula, particularly when agreement was achieved by using the surface velocity instead of the mean velocity, a procedure amounting to an arbitrary 25% correction.

It is difficult to conceive a formula for solids transportation that does not include the effect of size of particles carried. The variation with discharge of

[39] "Stable Channels in Erodible Materials," by E. W. Lane, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 126–127.

[40] "Beobachtungen ueber Geschiebefuehrung," by S. Kurzmann, Munich, A. Huber, 1919.

[41] "Der Geschiebetrieb und die Geschiebefracht," by A. Schoklitsch, *Wasserkraft und Wasserwirtschaft*, 1934, No. 4.

[42] Stauraumverlandung und Kolkabwehr," by A. Schoklitsch, Vienna, Julius Springer, 1935.

the particle size for channel stability has been demonstrated elsewhere[43] to be of appreciable magnitude.

The practical application of the Griffith formula (Equation (13)), necessitates ultimately a knowledge of the mean velocity in various verticals—an exceedingly difficult physical quantity to predict with reasonable accuracy.

Although the evidence is scientifically meager, the method presented in the paper seems to yield useful results. The approach is representative of the time-honored hydraulic method and stands in contradistinction to the contemporary technique of fluid mechanics as incorporated in the works of Messrs. W. Schmidt,[34] M. P. O'Brien,[35] and J. E. Christiansen,[36] on the subject of solids transportation. The writer leans toward the latter concepts in the belief that, in the end, they will be more fruitful. The contemporary experiments of fluid mechanics are broadening the language of hydraulics. Then, after a long period of development this new language, this new body of concepts, will be such a natural part of an engineer's intellectual equipment that he should be able to create and not merely conjecture with it.

Gerald Lacey,[44] Esq. (by letter).[44a]—Canal engineers in India who have made a study of silt transportation will read this paper with great interest. The writer was rather disappointed to find that the author had not modified his opinions expressed in 1927.[2] The writer had hoped that two contributions of his own on this subject[45] might have induced Mr. Griffith to reconsider his views. The part of Mr. Griffith's paper dealing with silt charge is of value as an interesting and original attempt at a quantitative solution.

When the author broadly refers to changes in the cross-sections of rivers and canals that could be computed mathematically by a "single elementary law" (see heading, "The General Theory") he is possibly representing the subject as being simpler than it really is. Furthermore, when Mr. Griffith proceeds to argue by reference to "Chezy's basic equations of flow" without expressing them mathematically, he is on debatable ground.

It is of interest to trace the subject historically. The classic equation is that proposed by Kennedy[5] in 1895,

$$V = 0.84\, d^{0.64} \qquad (22)$$

[43] "Bed-Load Transportation and the Stable-Channel Problem," by Samuel Shulits and W. E. Corfitzen, Jun. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 18th Annual Meeting, 1937, p. 465.

[34] "Die Massenaustauch in freien Luft und Verwandte Erscheinungen," von W. Schmidt, H. Grand, Hamburg, 1925.

[35] "Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Hydrology Section, 1933, p. 487.

[36] "Distribution of Silt in Open Channels," by J. E. Christiansen, *Transactions*, Am. Geophysical Union, Hydrology Section, 1935, p. 478.

[44] Superintending Engr., Irrigation Branch Secretariat, Lucknow, U. P., India.

[44a] Received by the Secretary September 16, 1938.

[2] "A Theory of Silt and Scour," by W. M. Griffith. *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927).

[45] "Stable Channels in Alluvium," by Gerald Lacy, *Minutes of Proceedings*, Inst. C. E., Vol. 229; and "Uniform Flow in Alluvial Rivers and Canals," *Loc. cit.*, Vol. 237.

[5] "Hydraulic Diagram for Canals in Earth," by R. G. Kennedy, *Minutes of Proceedings*, Inst. C. E., Vol. 119 (1895), p. 281.

Kennedy was of the opinion that in silt-transporting channels the bed was practically horizontal and the sides practically vertical. In Equation (22), therefore, V represents the mean velocity of the entire cross-section and d the average vertical depth as measured on the horizontal part of the bed. Kennedy described his equation as an empirical relationship, and although some attempt was made in his paper to deduce expressions having reference to silt charge, the equation was a practical engineer's empirical formula and of most value when used with discretion and applied to channels in India within the range of Kennedy's observations. Kennedy's paper was published without discussion by the Institution of Civil Engineers, London and, by the irony of fate, has been discussed almost without intermission ever since.

In 1912, Mr. W. J. Howley,[46] taking into account the fact that the side slopes in Madras, India, were not always vertical, preferred to deal with the mean vertical depth (the cross-sectional area divided by the water width) rather than the vertical depth, and produced for Madras the new equation,

$$V = 0.72\, d_m^{0.55} \qquad (23)$$

Both these equations are of importance inasmuch as they deal (very sensibly, in the writer's opinion) with the mean velocity, throughout a cross-section, and no attempt was made to draw inferences from such a generalized equation, as to the mean velocities in any given vertical.

In 1926, the writer[47] found that by treating the hydraulic mean depth, **R**, as a variable that the Kennedy equation could be replaced by the expression,

$$V = 1.16\, \mathbf{R}^{0.5} \qquad (24)$$

and, in 1930, demonstrated that both the Madras and the Kennedy data could be represented with accuracy by the general equation,

$$V = c\, \mathbf{R}^{0.5} \qquad (25)$$

In 1930 Mr. Griffith,[48] adopting the Kennedy equation as modified by Howley, and following the writer's expedient of reconciling the Kennedy and Madras data, obtained the expression for the mean velocity that is introduced in this paper as Equation (9).

From the practical engineer's viewpoint it makes little difference whether the writer's equation or that by Mr. Griffith is used. All the results that Mr. Griffith claims for his expression can be obtained as accurately and more readily by using the equation involving the hydraulic mean depth. B. A. Etcheverry, M. Am. Soc. C. E., has dealt with the subject competently and fully.[49]

The arguments advanced for using the mean vertical depth, d_m, rather than the hydraulic mean depth, **R**, as a variable, are shared by other engineers

[46] *Minutes of Proceedings*, Inst. C. E., Vol. 229, p. 261; "Critical Velocity Observations," Madras Public Works Dept., 1912.

[47] *Loc. cit.*, Vol. 223, p. 292.

[48] *Loc. cit.*, Vol. 229, p. 321.

[49] "Land Drainage and Flood Protection," by B. A. Etcheverry, 1931, see heading, "Problems of Flow in River Channels."

of the Kennedy school. Mr. Griffith, however, presents an original argument as to why the power of d_m should be greater than 0.5 in his equation.

The Chezy equation can be written in the form,

$$V = C \sqrt{R\,S} \qquad (26)$$

Experience has shown that the expression is incorrect. For regime channels there is now a great mass of evidence to show that for alluvial material of constant grade the equation,

$$V = C\,R^{0.75}\,S^{0.50} \qquad (27)$$

is approximately correct. That the Manning equation,

$$V = C\,R^{2/3}\,S^{1/2} \qquad (28)$$

is superior to the Chezy equation as originally presented, is conceded by every one.

Attempts to apply the Chezy general equation to calculating the mean velocities over verticals appear to the writer an unjustified step. There exists a certain mental confusion between equations of different types. Thus, when the writer first published his equation,[50]

$$V = C\,R^{0.5} \qquad (29)$$

for regime channels, it was argued[51] in India that the writer's equation was a backward step to Chezy. It was contended that the power in Chezy's equation (following Manning) was not $\frac{1}{2}$ but $\frac{2}{3}$ and, therefore, the power in the writer's equation was wrong. As a matter of fact there are two relationships, each one correct; namely: $V \propto R^{0.50}$; and, $V \propto R^{0.75}\,S^{0.50}$.

It follows, therefore, that in regime channels the slope varies inversely as the square root of the hydraulic mean depth, as the writer has demonstrated statistically.[52]

Mr. Griffith claims from Chezy's "basic equations of flow" that the average velocities in vertical planes above points on the bed should vary as the square roots of the vertical depths. If the original Chezy equation is used, it is wrong; if, on the other hand, Chezy as corrected (Manning, for example) is used, the power $\frac{2}{3}$ is greater than 0.57 and, therefore, Mr. Griffith's channels should perform in a manner opposite to that which he predicts. The point, however, is of little importance as the propriety of the entire line of argument is in doubt. Whether or not channels in incoherent alluvium widen depends on the simple relationship connecting the wetted perimeter, P, and the discharge,

$$P = 2.67\,Q^{0.50} \qquad (30)$$

a relationship first evolved by the writer[53] in 1928 and confirmed by thorough statistical analysis of data analyzed in Sind and the Punjab.[54] The value of the constant varies from 2.5 to 2.8, the power is invariably 0.50.

[50] "Stable Channels in Alluvium," *Minutes of Proceedings*, Inst. C. E., Vol. 229.

[51] *Indian Engineering*, Vol. LXXXIX, February 7, 1931, p. 111.

[52] *Minutes of Proceedings*, Inst. C. E., Vol. 237, p. 425.

[53] *Loc. cit.*, Vol. 229, p. 273.

[54] See Annual Repts., Central Irrig. Board Library, Simla.

There appears little doubt that the correct variable to use is the hydraulic mean depth, **R**, rather than the mean depth. It is true that silt is suspended by vertical components of eddies; but the section is generated by forces normal at all points to the wetted perimeter, and there is every reason why the hydraulic mean depth should be retained unless some other variable, without recourse to argumentation, can be proved statistically preferable.

Instead of Equation (11), the writer would use the equation,

$$\mathbf{R}_2 = \mathbf{R}_1 \left(\frac{P_1}{P_2}\right)^{2/3} \qquad (31)$$

Unfortunately, Mr. Griffith failed to quote (see "Application of Theory") the fundamental wetted perimeter, P, or the mean hydraulic depth, **R**. The writer would be grateful if Mr. Griffith would supply the missing values of P and **R** and also compare and quote the accuracy of the result obtained by using Equation (31).

The theory offered by Mr. Griffith in respect of silt charge is original and valuable. To the writer it appears that the theory would be improved by using his own simple equation: Silt factor $\infty \frac{V^2}{\mathbf{R}}$.

The silt in suspension is a function of the turbulence, or as V. V. Tchikoff, M. Am. Soc. C. E., terms[55] it, "kineticity." For this turbulence Mr. Griffith presents the ratio, $\frac{V}{d_m{}^{0.57}}$, instead of the writer's $\frac{V}{\mathbf{R}^{0.50}}$.

The ratio, $\frac{V^2}{\mathbf{R}}$, has the dimensions of an acceleration and has a physical meaning associated with turbulence, or "vorticity," whereas the expression, $\frac{V}{d_m{}^{0.57}}$, is dimensionally unsatisfactory and, from a practical standpoint, somewhat unmanageable.

With much of Mr. Griffith's general views the writer is in agreement, but he feels that little progress can be made in fundamental theory by retaining the mean depth as a variable or by relying too implicitly on a doubtful interpretation of Chezy's equation.

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[55] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 167.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE THREE-POINT PROBLEM IN A CO-ORDINATED FIELD

Discussion

BY JOHN R. JAHN, ASSOC. M. AM. SOC. C. E.

JOHN R. JAHN,[5] ASSOC. M. AM. SOC. C. E. (by letter).[5a]—In bringing this valuable graphical device into a more exact form for use in balanced surveys, the author is to be congratulated. His method is ingenious and should be of considerable value in the work of the larger surveying organizations that employ men who are sufficiently skilled in the mathematics of geodesy so that errors of manipulating the formulas, and errors of technique in the analysis, do not defeat the straightforward solution of the problems. In the illustrative example, the basic data, with the exception of that applying to Station *O*, are of uniformly high accuracy. Had this not been the case, the analysis would probably have failed, due to ambiguity in the many sets of possible locations for Station *P*. The writer has spent much time attempting to find single points that would satisfy the directional data for more than three-point groups. Part of the trouble was caused by slightly inaccurate angular data and part by slightly inaccurate station positioning. The author's method of working toward the most reasonable position by the use of unit bands was attempted without success, although the finer analysis that Mr. Rowe uses was not then known to the writer.

In the example, the observed angles were adjusted to seconds. This indicates the use of an instrument more accurate than the ordinary engineer's transit, reading to minutes. Multiple pointings can be taken and the average value used. However, to be of use to the average trained surveyor, the writer believes that a simpler method of analysis is necessary to enable the surveyor to add this "three-point problem" to his mental equipment. For several years, the writer has been attempting to formulate a procedure, using the mathematics of analytic geometry in co-ordinated field data, in order that field survey points might be referenced to established landmarks, such as flag-poles,

NOTE.—This paper by R. Robinson Rowe, M. Am. Soc. C. E., was published in June, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by Messrs. Oscar S. Adams, and George D. Whitmore.

[5] Civ. Engr., Riverside, Calif.

[5a] Received by the Secretary October 13, 1938.

steeples, windmills, etc., whose "positions" could be determined closely. This is particularly valuable in open country, where near-by reference points are not always at hand. With a transit line tied into co-ordinated landmarks by three-point or multiple-point directional data, the recovery of old stations would be considerably aided.

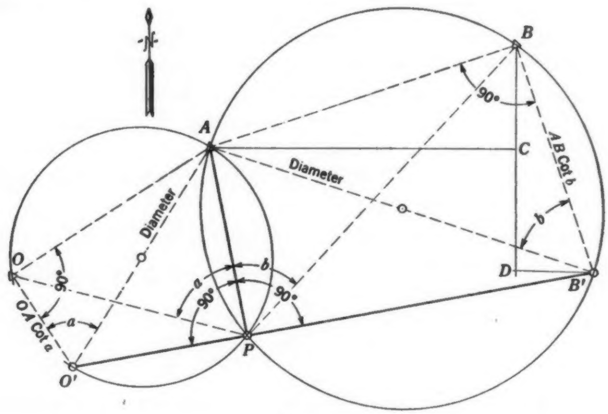

FIG. 4

In Fig. 4 which is similar to Fig. 1, Point P lies at the intersection of the two determining circles passing through Points O and A and through Points A and B. By constructing Point O' such that $\overline{OO'} = \overline{OA} \cot a$, Circle $O\ A\ P$ is divided by Diameter $A\ O'$, since $O'\ O\ A$ is a right angle and Chord $O\ A$ subtends Angle a; hence, Angle $A\ P\ O'$ is a right angle. By like construction $\overline{BB'} = \overline{AB} \cot b$, and likewise Angle $A\ P\ B'$, is a right angle. Hence, $O'\ PB'$ is a straight line and Point P is located at the intersection of two lines, one through Points O' and B', the "closing line" of a traverse having a slope,

$$m = \frac{dy}{dx} = \frac{\text{Difference of northings}}{\text{Difference of eastings}} \ldots\ldots\ldots\ldots\ldots (22)$$

and the other through Point A and having a slope at right angles to the former. If Line $O'\ B'$ is expressed by the equation,

$$y = m\,x + c_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23a)$$

and the line through Point A is expressed by the equation,

$$x = -\,m\,y + c_A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23b)$$

the two equations may be solved for x_P and y_P from the combination; that is, $x_P = -\,m^2\,x_P - m\,c_B + c_A$, or,

$$x_P = \frac{-\,m\,c_B + c_A}{1 + m^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

This would be laborious if the lengths of Lines $O\ A$ and $A\ B$ were to be computed as the square root of the sum of the squares of co-ordinate differences.

However, Triangles $A\ C\ B$ and $B\ D\ B'$, representing the longitude and latitude differences of Lines $A\ B$ and $B\ B'$, are similar to each other since corresponding angles are equal. Then, $\frac{\overline{AC}}{\overline{BD}} = \frac{\overline{BC}}{\overline{DB'}} = \frac{\overline{AB}}{\overline{BB'}}$; but $\overline{BB'} = \overline{AB} \cot b$, so that "difference of northings, $\overline{BB'}$" (or BD) = "difference of eastings, $\overline{AB}$" $\cot b$ (or $\overline{AC} \cot b$); and, "difference of eastings, $\overline{BB'}$" (or $\overline{DB'}$) = "difference of northings, $\overline{AB}$" $\cot b$ (or $\overline{BC} \cot b$). It follows that Line $O'\ B'$ is the closing line of a traverse, $O\ O'$, $O\ A$, $A\ B$, and $B\ B'$, and the signs of the several courses can be visualized and set down without likelihood of error. It is to be noted that this analysis is the same as that leading to Equation (7). Fig. 4 shows Angles a and b as acute, or less than 90 degrees. If either or both had been obtuse, or greater than 90°, Lines $O\ O'$ or $B\ B'$ would have extended on the opposite sides of Line $O\ A$ or Line $A\ B$ from Point P.

A work sheet using the author's data for Signals D, E, and F is illustrated by Table 4. Let $m = \frac{-\ 11\,991.9}{-\ 15\,699.1} = +\ 0.763859 = \cot 52° \ 37\frac{1}{2}'$; from which the true bearing of Line $D\ P$ is $270° + 52°\ 37\frac{1}{2}' = 322°\ 37\frac{1}{2}'$ (as read, $322°\ 37\frac{1}{4}'$; and, therefore, the correction to the angles is $+\ \frac{1}{4}'$).

TABLE 4.—COMPUTATION FORM FOR ALTERNATE SOLUTION

Signal (See Fig. 2)	Azimuth	Included angle	Differences of: Northings	Differences of: Eastings	Co-ordinates: North	Co-ordinates: East	Remarks
F'.....	....				....	....	Not required
		....	−413.3*	+5 837.8			
F.....	194° 24′.5				−20 033.7	+5 941.4	Given
		128° 12′.75	+7 415.1	+525.0			
D.....	322° 37′.25				−12 618.6	+6 466.4	Given
		137° 24′.75	−2 366.4	+6 761.8			
E.....	100° 02′				−14 985.0	+13 228.2	Given
		....	+7 356.5	+2 574.5			
E'.....					−7 628.5	+15 802.7	From Signal E
....	....	....	−11 991.9	−15 699.1	....	....	To balance

* $413.3 = 525 \times \cot 128°\ 12\frac{3}{4}'$.

At Point E', Equation (23*a*), for Line $E'\ F'$, yields: $y = +\ 0.763859\ x - 19\,699.5$ ($c_{E'} = -\ 15\,802.7 \times 0.763859 - 7\,628.5$).

At Point D, Equation (23*b*) for Line $D\ P$ yields $x = -\ 0.763859\ y - 3\,172.4$; and, solving for x, $x = -\ 0.58348\ x + (-\ 0.763859 \times -\ 19\,699.5) - 3\,172.4 = \frac{+\ 11\,875.2}{+\ 1.58348} = +\ 7\,499.5$; and, $y = (+\ 0.763859)(+\ 7\,499.5) - 19\,699.5 = -\ 13\,971.0$ (see Case I, Fig. 5). A form sheet for the foregoing computation (see Table 4) makes the solution straightforward, and with the aid of natural functions and a computing machine, the computation can be made in 5 min.

As the author states, the analysis of data on multiple pointings involving more than three stations brings in many possible solutions. It will be noticed that in the foregoing analysis any station (D, for example) may be taken for the central, or reference line. If perpendiculars are taken, and auxiliary points, such as E', etc., are computed for each pair of points (such as ED), a series of

such auxiliary points will range along the true line passing through Point P and normal to Line DP. Using the author's data Table 5 gives such results. A first approximation is the "range-line," C' E':

TABLE 5.—STATIONS REFERRED TO STATION D, CO-ORDINATES; N − 12 618.6; AND, E + 6 466.4

To Station	Included angle	dN	dE	$d'N$	$d'E$	Co-Ordinates	
						North	East
E.......	137° 24′ 43″	−2 366.4	+6 761.8			−14 985.0	+13 228.2
E'......				+7 356.5	+2 574.5	− 7 628.5	+15 802.7
F.......	231° 47′ 13″	−7 415.1	−525.0			−20 033.7	+5 941.4
F'......				+413.3	−5 837.9	−19 620.4	+103.5
O.......	240° 41′ 43″	−13 022.9	−3 996.9			−25 641.5	+2 469.5
O'......				+2 243.4	−7 309.5	−23 398.1	−4 840.0
A.......	276° 13′ 35″	−16 069.2	−23 316.5			−28 687.8	−16 850.1
A'......				−2 543.8	+1 753.2	−31 231.6	−15 096.9
B.......	304° 44′ 23″	−1 799.0	−8 663.1			−14 417.6	−2 196.7
B'......				−6 007.5	+1 247.5	−20 425.1	−949.2
C.......	312° 24′ 48″	+1 031.5	−26 010.1			−11 587.1	−19 543.7
C'......				−23 761.6	−942.3	−35 348.7	−20 486.0

Co-Ordinates C'..........	− 35 348.7	− 20 486.0
Co-Ordinates E'..........	− 7 628.5	+ 15 802.7
Change.................	+ 27 720.2	+ 36 288.7

and, the slope, m, $= \dfrac{+\ 27\ 720.2}{+\ 36\ 288.7} = +\ 0.763880.$

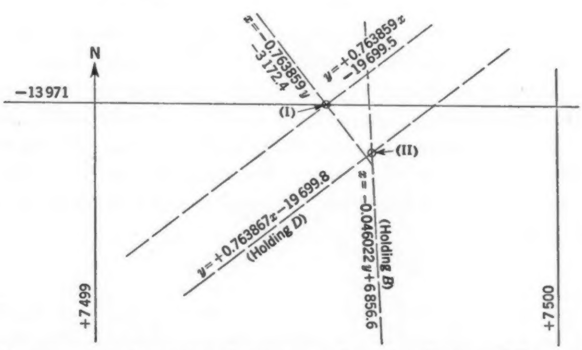

FIG. 5.—SOLUTION BY ANALYTIC GEOMETRY: CASE I = SIMPLE 3-POINT FIX; AND CASE II = BALANCED 6-POINT FIX

Using the co-ordinates of Point E', the equation of the "range-line" is,

$$y = +\ 0.763880\ x - 19\ 699.9 \qquad (25)$$

When the values of x for each of the auxiliary points is substituted in Equation (25) and the y-value computed, the departure of the value in Table 5 can be ascertained. Since the line passes through Points E' and C' these departures

are zero. The other departures are F', $+ 0.4$; O', $- 1.0$; A', $+ 0.5$; and, B', $- 0.1$. The departure of the value for O' is considered as excessive and is rejected. A new line may be chosen that will more nearly equalize the departures of the tabulated data. Such a line would pass 0.3′ north of Point C' and 0.1′ south of Point E'. This can be seen on a graph of exaggerated scale (Fig. 6)

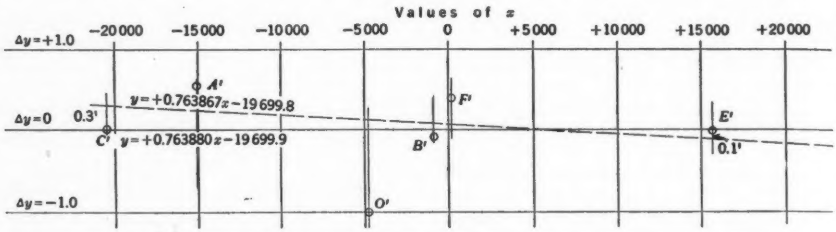

FIG. 6.—POSITION OF AUXILIARY POINTS WITH RESPECT TO "RANGE-LINE"

in which "x (East)" is plotted: 1″ = 5 000 ft; and, "difference of y (North)" = 1″ = 1 ft. The new equation for the "range-line" would then be,

$$y = + 0.763867\, x - 19\,699.8 \qquad (26)$$

Since Station D may have been in error, it will be necessary to choose another "range-line," a quadrant removed from the pointing to D. Consider, for example, a set of data referred to Station B and determine a range line that will distribute the computed errors approximately equally on either side. An analysis gives a line normal to Line $P\,B$ having the equation,

$$x = - 0.046022\, y + 6\,856.6 \qquad (27)$$

and the solution of the two equations for x and y (Case II, Fig. 5) gives the probable position of the point as: North $- 13\,971.1$; and, East $+ 7\,499.6$.

The writer feels that the careful use of the "three-point-problem" can have a distinct value in giving an independent check on survey work, particularly in certain types of reconnaissance work where it may not be economically possible to loop the survey into closed traverses. For this purpose it would be necessary, of course, to refer the co-ordinates of the basic control stations to the meridian of the initial station.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

LATERAL EARTH AND CONCRETE PRESSURES

Discussion

By Messrs. Robert G. Hennes, Robert F. Legget, and Charles Terzaghi

Robert G. Hennes,[12] Assoc. M. Am. Soc. C. E. (by letter).[12a]—It is important to realize that, as the authors indicate, Equations (1) to (11) are primarily intended for the design of sheeting and bracing. When applied to permanent rigid structures, such as underground tanks and other sub-surface construction, in soils subject to elastic expansion and contraction with seasonal variations in water content, the value of K in Equation (11) is not necessarily within the range of 0.6 to 0.2, but may reach much higher values.

When gravity is the sole body force, F, in Equation (1), becomes equal to s_x, the stress in the soil. If a sufficiently prolonged drought causes shrinkage, the surface tension of the capillary water creates a capillary pressure of intensity, p_c, throughout the soil adjacent to the substructure, and the horizontal earth pressure is,

$$F' = - s_x + p_c \qquad (38a)$$

in which $s_x = (p_c + w\,y)\left(\frac{\mu}{1 - \mu}\right)$; or,

$$F' = p_c - (p_c + w\,y)\left(\frac{\mu}{1 - \mu}\right) \qquad (38b)$$

If F' falls below the magnitude of the minimum earth pressure, F_{II}, acting on a yielding wall, as determined by methods applicable to Phase II, then the soil mass behind the wall will slip until $F' = F_{II}$ as a possible minimum value. When the water-table is again raised by rainfall the tendency of the soil to expand with falling capillary pressure is limited by the fact that the soil as a whole has slipped into closer contact with the rigid structure. The ultimate

Note.—The paper by Lazarus White and George Paaswell, Members, Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by A. E. Cummings, M. Am. Soc. C. E.

[12] Asst. Prof., Civ. Eng., Univ. of Washington, Seattle, Wash.

[12a] Received by the Secretary October 17, 1938.

pressure in this case is not the value obtained from Equation (1), but:

$$F = -s_x = -\left[(p_c + w\,y)\left(\frac{\mu}{1-\mu}\right)\right] - w\,y\left(\frac{\mu}{1-\mu}\right) + F_{\text{II}} \quad \text{(39a)}$$

or,

$$F = F_{\text{II}} - K\,(p_c + 2\,w\,y) \quad \text{(39b)}$$

The value of F, as thus determined, cannot rise above the value of the passive earth pressure, as it would then be relieved by the rupture of the soil mass. However, this latter criterion probably imposes a limitation seldom attained in Nature, as the wet-dry cycles are not generally of sufficient amplitude to make the actual capillary pressure in cohesive soils a maximum; that is, the subsoil is not commonly dried to its shrinkage limit for any appreciable depth.

Equation (39*b*) is introduced mainly to emphasize the distinction between temporary and permanent rigid structures. Its application in a specific case would be handicapped by the need to determine the proper value of p_c. Where circumstances warrant the procedure, this obstacle could be overcome by measuring the natural moisture content of the soil at various times of the year, and then finding the corresponding pressure from a consolidation test on an undisturbed sample.

If it is important to distinguish between a permanent, rigid, underground structure and a temporary structure (such as sheeting and bracing) in choosing a basis for design, it is also important to realize that neither method would ordinarily be chosen for the design of retaining walls, which fall within the boundaries of Phase II of the authors' "Introduction." For reasons of economy most engineers will probably continue to use the classical theories in those cases where a slight yielding of the wall is of no great consequence. Such a procedure is subject to the restriction that the wall must be actually free to slide or otherwise deflect the necessary amount. Where this rule is violated (as often happens in the case of right-angle turns in the low retaining walls on residential property), the wall will crack unless Equation (39*b*) has been satisfied.

It is possible, of course, to use Equation (11) for the design of flexible retaining walls, by choosing an arbitrary value for K, lower than that furnished by Equation (1). When so used, Equation (11) is no longer based upon the theory of elasticity, but becomes a statement of the equivalent fluid method. The limitations of this and other methods with respect to the position and direction of the resultant pressure make it desirable to consult the reference[4] given by the authors for cases involving Phase II.

ROBERT F. LEGGET,[13] ASSOC. M. AM. SOC. C. E. (by letter).[13a]—It is refreshing to encounter so forthright a paper on so old and contentious a subject as lateral earth pressure. The background of the ten years of work

[4] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., *Paper J-3*, Vol. 1, Harvard Univ., June 22, 1936.

[13] Asst. Prof., Dept. of Civ. Eng., Univ. of Toronto, Toronto, Ont., Canada.

[13a] Received by the Secretary October 24, 1938.

which the authors mention might be thought to have provided practical evidence in support of the conclusions advanced. Fig. 5 appears to be the only such record included in the paper. Of the fourteen pages, nine are devoted to an interesting mathematical discussion of the effect of surface loads on lateral pressure, and only two to the general problem of design encountered in the vast majority of lateral earth-pressure problems. The "Theory of Design" appears to be contained in Equation (11), with the accompanying qualification, which gives the equivalent hydrostatic pressure for some arbitrary value of K, as now commonly used.

These features of the paper are mentioned since they contrast strangely with statements regarding fundamentals such as: (*a*) "* * * it has been found that the general shape of such [soil] fracture is as shown in Fig. 1. * * * [which] is independent of the type of ground * * *." (See heading, "Phase I.") And (*b*) "[The authors] have recognized that these terms are misleading and of no significance in discussing lateral pressures. * * * [and] that the road to correct determination of soil phenomena lay in such application and not in futile attempts to extend the classic formulas * * *." (See heading, "Conclusions"; the "terms" include coherent, non-coherent, internal friction, etc.). These two statements defy discussion; they prompt many questions, typical of which the following are submitted:

(1) What proof can the authors offer that the general shape of earth fracture behind a retaining wall is always curved and independent of the type of ground?

(2) Professor C. F. Jenkin[14] has conducted an investigation on the lateral pressure exerted by sand, in which theory and experience were correlated accurately. Are investigations of this caliber included in the general group of "futile atempts to extend classic formulas," and if not, how can they be considered in relation to the statement referred to in the first question?

CHARLES TERZAGHI,[15] M. AM. SOC. C. E. (by letter).[15a]—In their paper the authors reject the application of the classical earth-pressure theories to the computation of the pressure of earth on the timbering of cuts. They also propose the application of the formula of Boussinesq to the computation of the lateral pressure due to surcharges.

During the past few years the classical earth-pressure theories have been superseded by much broader concepts, involving the elimination of the traditional assumptions of hydrostatic pressure distribution and of plane surfaces of sliding. As a consequence, most of the contradictions between theory and experience have disappeared. This "general wedge theory" explains both the high location of the center of the pressure on the timbering of cuts and the concentration of the pressures in the vicinity of the supports of flexible members. The only requirement for the successful application of the general wedge theory consists in almost full mobilization of the shearing resistance of the soil along the potential surface of sliding.

[14] "The Pressure on Retaining Walls," by C. F. Jenkin, *Minutes of Proceedings*, Inst. C. E., Vol. 234, 1932, pp. 103–223.

[15] Dr. Ing.; Graduate School of Eng., Harvard Univ., Cambridge, Mass.

[15a] Received by the Secretary November 3, 1938.

In the earth-pressure tests on clean sand, reported by the writer[16] in 1934, it was found that the minimum lateral pressure developed before the wall yielded, exceeded one-fifth of the yield required for inducing a slip in the back-fill. No minimum pressure is conceivable unless the shearing resistance of the back-fill is mobilized. Hence, the fundamental requirement of the general wedge theory was satisfied long before a noticeable slip in the back-fill occurred. For the same reason one is entitled to expect that the shearing resistance of the banks on both sides of a cut through fairly dense, sandy soil can be almost fully mobilized, although the banks are still far from failing along a visible shearing surface. In such a case the application of the general wedge theory would also be justified. Quite recently this conclusion was confirmed by the results of reliable measurements made by the Siemens Bau-union in Berlin, Germany, in a cut through sand and fine gravel. On the other hand, the lateral pressure of a plastic clay on the timbering of a cut will undoubtedly be greater than the minimum pressure computed by means of the general wedge theory. For dealing with cases of this type, the authors recommend the theory of elasticity. In order to accept their suggestion, the investigator would need to know: (*a*) How to determine the value of the required soil constants by experiment; (*b*) how to compute the lateral pressure from the soil constants; and (*c*) how the results thus obtained compare with experience. No answer to these vital questions can be found in the paper. Hence, it would be premature to discuss the practical value of the suggestion.

The proposed application of Boussinesq's formulas to the computation of the lateral pressure, due to surcharges, is so tempting that the writer succumbed to this temptation twenty years ago, at the time when he started to invade his present field of research.[17] In the reference the authors can even find their Fig. 3 and the pertinent computations. However, the writer never recommended his theory for practical application because he realized at the very outset that the actual pressure values can either be much smaller or considerably greater than the theoretical ones, depending on the conditions of lateral support for the surcharged bank of earth. Thus, for instance, if the surcharge consists of a mass of earth rising at a slope from the crest of a retaining wall, the lateral pressures computed by means of Boussinesq's formulas were found to be far in excess of those which were measured by Audé and many others. On the other hand, if one applies the theory to the computation of the lateral pressure produced by concentrated surcharges whose weight is small compared to that of the "sliding wedge," the real values are approximately twice as high as the theoretical ones. It may suffice to call attention to the results of pertinent experimental investigations by M. G. Spangler,[18] Assoc. M. Am. Soc. C. E., and to the interpretation of the test results by R. D. Mindlin,[19] Jun. Am. Soc. C. E.

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[16] "Large Retaining Wall Tests," by Charles Terzaghi, *Engineering News-Record*, February 1, 1934.

[17] "Die Erddruckerscheinungen in örtlich beanspruchten Schüttungen," von Karl Terzaghi, *Öst. Wochenschrift für den öff. Baudienst*, 1919.

[18] "Earth Pressures on a Retaining Wall Due to a Concentrated Surface Load," by M. G. Spangler, *Proceedings*, First International Conference on Soil Mechanics, Vol. I, Cambridge, Mass., 1936, p. 200.

[19] "Pressure Distribution on Retaining Walls" (Discussion), by R. D. Mindlin, *Proceedings*, International Conference on Soil Mechanics, Vol. III, Cambridge, Mass., 1936, p. 155.

In 1929, E. Gerber published the results of direct measurements of the lateral pressure produced by circular and rectangular surcharges resting on the surface of a back-fill with a depth of 80 cm.[20] The measurements were made by means of very sensitive pressure cells. The writer suggests that the authors should check their theory against Mr. Gerber's test results, but he is afraid that the outcome will be rather disappointing.

The lateral surcharge pressure as computed by means of Boussinesq's formulas can be intolerably far from the truth, therefore, and the error can either be positive or negative, depending on the degree of rigidity of the lateral support and on the ratio between the weight of the surcharge and that of the "sliding wedge." On account of the difficulties connected with pertinent theoretical investigations, the subject still offers a promising field for experimental research. It would be particularly useful to measure the influence of the weight of spoil-banks and of other surcharges on the pressure in the struts of cuts at different depths below the surface.

If results of actual measurements and observations in the field are available, it is to be hoped that the authors will include them in the closing discussion.

[20] "Untersuchungen über die Druckverteilung in örtliche belastetem Sand," von E. Gerber; pub. Doctor's Thesis, Zürich, Switzerland, 1929.

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DISCUSSIONS

TRANSPORTATION OF SAND AND GRAVEL IN A FOUR-INCH PIPE

Discussion

By Messrs. Fred R. Brown, Josef E. Montgomery, Elliott J. Dent, and David L. Neuman

Fred R. Brown,[8] Jun. Am. Soc. C. E. (by letter).[8a]—Despite the findings of recent worth while investigations, the information on the transportation of material in pipe lines is still rather meager. Hence, there are not at hand data sufficient for the development of a formula with comprehensive application to the movement of solids through pipe lines. If such a formula can be determined its development can follow only upon the collection of further data, and the correlation of these data with those previously obtained.

TABLE 3.—Variations in Friction Factor, f, for Changes in Solids Concentration in a Thirty-inch Pipe

Velocity, V, in feet per second	Values of f for the Following Percentages of Solid Matter:			Velocity, V, in feet per second	Values of f for the Following Percentages of Solid Matter:			Velocity, V, in feet per second	Values of f for the Following Percentages fo Solid Matter:		
	Clear water	10	20		Clear water	10	20		Clear water	10	20
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
4.......	0.030	....	0.265	8.....	0.025	0.052	0.074	12.....	0.022	0.029	0.043
5.......	0.029	....	0.175	9.....	0.025	0.043	0.063	13.....	0.022	0.028	0.039
6.......	0.027	....	0.125	10.....	0.024	0.038	0.053	14.....	0.021	....	0.037
7.......	0.026	....	0.094	11.....	0.023	0.032	0.047	..	....	....	....

It is with this point in mind that the computations in Table 3 are offered. The data for these computations were taken from the curves of the dredge, *Alpha*,[9] presented in the discussion by Miss Blatch[7] and bear out Mr. Howard's

Note.—The paper by J. W. Howard, Jun. Am. Soc. C. E., was published in September, 1938, *Proceedings*. This discussion is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion of the paper.

[8] Jun. Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

[8a] Received by the Secretary October 24, 1938.

[9] "Dredges and Dredging on the Mississippi River," by the late J. A. Ockerson, Past-President, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XL (December, 1898), p. 215.

[7] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 406.

findings on 1-in., 3-in., and 4-in. pipes in that the friction factor in pipes transporting material increases with the percentage of material and decreases with the velocity. These values of f may be incorporated in Table 2, compiled by Mr. Howard, thus presenting f-values in 1-in., 3-in., 4-in., and 30-in. pipes. All values in Table 3 are computed from a head that is in feet of mixture rather than in feet of water.

The magnitude of velocities and the percentage of solids used in deriving the foregoing f-values were determined by the volumetric method. Velocities obtained by other methods (salt solution, gage stick, etc.), for pipes transporting material not in suspension, are not true average velocities. Hence, such velocities will introduce appreciable errors in any computations involving velocity. Likewise, there are numerous ways of determining the percentage of solids pumped, but percentages determined by volumetric measurement are the most accurate. Errors in velocity or the percentage of solids will increase as the percentage of solids increases, if other than the volumetric method of measurement is used. For this reason, any data that might be incorporated with those of Mr. Howard in an attempt to arrive at a formula for pipes transporting material should have the velocities and the percentages of solids determined volumetrically. Data obtained otherwise would not be comparable with that presented so ably by Mr. Howard.

JOSEF E. MONTGOMERY,[10] ESQ. (by letter).[10a]—One of the most interesting points developed by Mr. Howard is that which shows the increase in values of Darcy's f with decrease in velocity for flow in a 4-in. pipe carrying water and entrained sand or gravel. Below certain values, the relation of f to velocity is much different for solids-laden water than for clear water. The cause of this difference is shown by comparison with f-values for flow of clear water in similar pipes. Data suitable for such comparison are found in the results of previous work[11] by Fred C. Scobey, M. Am. Soc. C. E., published in 1930. Using the data obtained by Mr. Scobey, curves have been plotted in Fig. 9 to show the relation of values of f in Equation (2) to mean velocity for the flow of clear water in a 4-in. pipe. Representative data obtained by Mr. Howard have been likewise plotted. At higher velocities all values approach the same relation to velocity, although the scope of the investigation was not sufficient to provide for full comparison.

The curves for the runs with gravel are least similar to the curves for clear water, but Mr. Howard states that some gravel remained motionless on the bottom during the full range of velocities observed. Curves for the sand approach those for clear water, and two of the curves coincide at higher velocities. It is apparent that the highest values of f occur for the lowest velocities and highest solids concentration, in which some solid material was observed to be at rest on the bottom of the pipe. The relation of f-values to velocity for sand-laden water approaches that for clear water in those runs in which all material was known to travel in suspension.

[10] Engr. Aide, U. S. Waterways Experiment Station, Vicksburg, Miss.

[10a] Received by the Secretary October 24, 1938.

[11] "The Flow of Water in Riveted Steel and Analogous Pipes," by Fred C. Scobey, *Technical Bulletin, No. 150,* U. S. Dept. of Agri., Washington, D. C., pp. 35 and 38.

The data show that entrainment of solids (that is, sand or gravel) is not alone sufficient to produce appreciable effect on the variation of f with velocity, but that it is deposition of such material at the bottom of the pipe that affects the relation. This change is not due to the pressures created by constriction of cross-sectional area, as stated by Mr. Howard under "Head Loss Variations," since the friction factor, f, is generally unaffected by pressure changes. The explanation is found in the fact that values of f are very sensitive to any deviation from the nominal area value used in the computation of this coefficient.

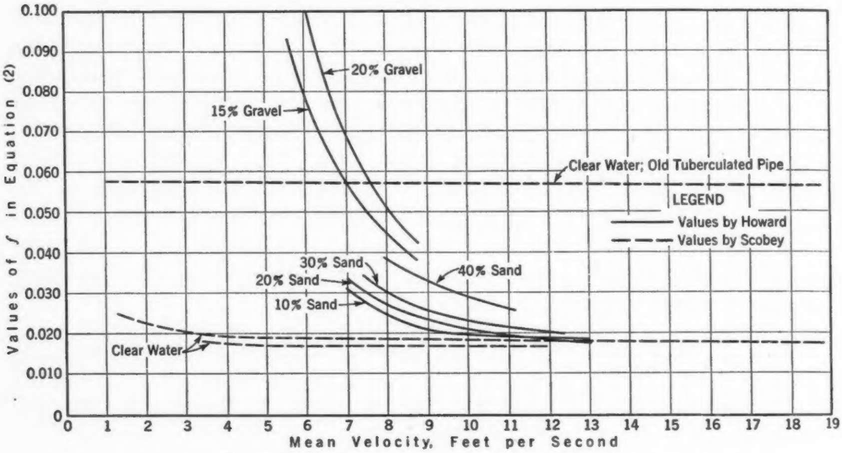

FIG. 9.—COMPARATIVE VALUES OF DARCY'S f.

Comparisons of this type can be put to practical use in dredging and similar operations because they indicate the possible existence of motionless solids on the bottom of a pipe and the consequent waste of energy. Curves of the type shown for gravel in Fig. 9 show the excessive energy loss due to insufficient velocity for that particular concentration. If extended by additional data, these curves would indicate further the optimum velocity for transportation of such material.

ELLIOTT J. DENT,[12] M. AM. SOC. C. E. (by letter).[12a]—This paper is of particular interest to the writer as it supplies a more complete picture of the manner in which sand and water mixtures flow through pipe lines than any of the reports that had previously come to his attention. With the information presented, combined with that previously available, and combined further with data furnished by Mr. Howard by letter, it is possible to picture the behavior of quite a wide range of mixtures flowing in a 4-in. pipe. Although such a portrait may not be applicable to full-sized dredge pipes in a quantitative sense, the data do afford a qualitative picture for the benefit of dredge designers and operators. These laboratory results will also constitute an excellent guide for the preparation of a program for further investigations.

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[12] Col., U. S. Army (*Retired*), Washington, D. C.

[12a] Received by the Secretary October 27, 1938.

The author emphatically calls attention to the effect of the size of the material handled, and gives an analysis of the sand and gravel used in the experiments reported upon. The importance of this feature cannot be overestimated and it is to be hoped that, in all future investigations, this practice will be continued. In this paper, as in most others on the subject, the friction losses due to the flow of water only have been inadequately described.

In a relatively unexplored field of this nature it is desirable that the factual data as well as the adjusted results be published. Students of the subject should be free to use their own methods for adjusting, compiling, and presenting the facts and the conclusions derived therefrom. For this subject there is no standardized system of presentation and, pending the time when a satisfactory system may be evolved, a wide variety of viewpoints is most desirable.

In this and other papers on the subject references are made to the manner in which the pipe lines are completely blocked off when the velocity falls below a certain value. The author has very kindly furnished the following data showing the blocking-off points in sand tests of a 4-in. pipe, for some of the mixtures handled:

Velocity	Percentage of sand concentration
5.7	26
7.0	27
7.5	34
8.2	52

In adjusting these data the writer has assumed that another point would correspond to a velocity equal to ∞ and a percentage of solids equal to ∞. The points were plotted and a smooth curve was drawn conforming as closely as possible to the data. The values for 15%, 20%, and 25%, were then taken from the curve. It is hoped that, in future investigations, the blocking-off points will be carefully established.

In presenting his adjusted data the author has reduced the losses of head to terms involving the specific gravity of the mixtures handled and has computed and tabulated the values of f in the Darcy formula. This method does not afford a simple or readily visualized picture of the case.

A formula for the loss of head, which is better known than Equation (1) of the paper, is the Chezy formula:

$$V = C\sqrt{R\,S} \qquad (6)$$

in which R = the mean hydraulic radius $= \frac{D}{4}$, in feet; S = the sine of the angle of slope $= \frac{h_f}{L}$; and, C = a factor often erroneously referred to as a constant.

Combining Equations (1) and (6):

$$V^2 = \frac{2\,g\,D\,h_f}{f\,L} = C^2\,R\,S \qquad (7)$$

Substituting S for its equivalent, $\frac{h_f}{L}$, and R for its equivalent, $\frac{D}{4}$:

$$V^2 = \frac{8\,g}{f}\,R\,S = C^2\,R\,S \qquad (8)$$

Therefore, $\frac{8\,g}{f} = C^2$; and, $f = \frac{8\,g}{C^2}$. It is obvious that phenomena that cause a variation in the values of C in the Chezy formula will cause corresponding variations in the values of f in the Darcy formula.

It is well known that, in the Chezy formula, the values of C vary with the slope, hydraulic radius, and roughness of the inner surface of the pipe. About two generations ago a formula for computing the value of C was devised and is known as the Kutter formula. It is laborious to solve and many tables and diagrams have been published to assist in this process. Similar tables and diagrams can be prepared for finding the values of f in the Darcy formula if it is desired to do so.

In dealing with the flow of water through pipe lines, the value of f is a complicated function of roughness, diameter, and slope. In dealing with the flow of mixtures of sand and water, the author has added to the complications by making f a function of roughness, slope, diameter of pipe, coarseness of material, percentage of material in the mixture, and specific gravity of the mixture.

When dealing with water alone the values of C in the Chezy formula were so involved that a simplified expression was urgently needed. About a generation ago the late Allen Hazen and the late Gardiner S. Williams, Members, Am. Soc. C. E., devised the following exponential formula for computing the friction due to the flow of water in pipe lines. The nomenclature used is the same as that in Equation (6):

$$V = C\,R^{0.63}\,S^{0.54}\,0.001^{-0.04} \qquad (9)$$

A slide-rule was devised for the convenient and rapid solution of Equation (9), and tables and diagrams have been prepared and published in handbooks and elsewhere. The writer uses this method for computing the losses of head in pipe lines. Mr. Howard states that, for water only, the exponential formulas are to be preferred; but for mixtures of sand and water—a more complicated condition—he uses the Darcy formula.

The normal problem confronting the dredge designer or operator is to determine the loss of head due to a given velocity when the other items of the problem have been determined or assumed. The author calls attention to the difficulties encountered in attempting to devise an exponential equation or equations that would give reliable results.

In Fig. 6 Mr. Howard has plotted certain data, and it is believed that an extension of this graphic method may result in a very useful diagram. If existing information is plotted now, the diagrams will be of immediate use to designers and operators. The diagrams will also indicate the nature of the information that should be secured by additional investigations in order to increase the sum total of knowledge on the subject.

In Fig. 10 the writer has plotted certain fairly reliable information with respect to the friction losses due to the flow of various concentrations of various kinds of materials, at various velocities, in a 4-in. pipe. Some adjustments of the basic data have been necessary. The curve for water only has been based on the value, $C = 140$, in the Williams-Hazen formula. This conforms fairly well with the author's observation that $f = 0.0206$ when the velocity was 5.4, and $f = 0.0164$ when the velocity was 11.2.

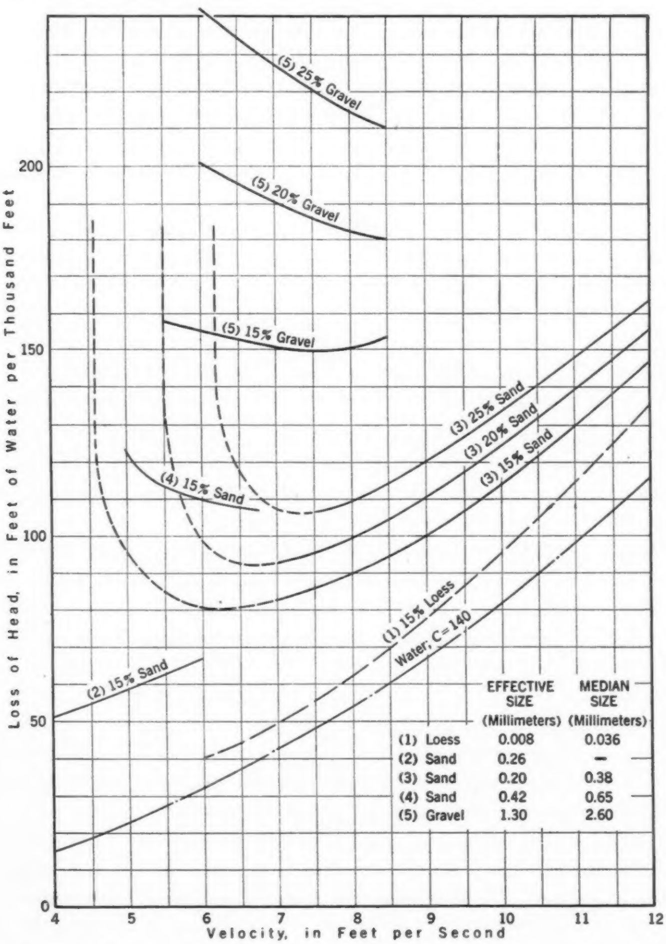

FIG. 10.—FRICTION IN A FOUR-INCH PIPE; MIXTURES OF SILT, SANDY GRAVEL, AND WATER

Curve (1), 15% loess, was plotted from data supplied by the author. The effective size (10% of the material is finer than this size) was taken from a diagram also furnished by Mr. Howard. Curve (2), 15% sand, was obtained from data published by John A Vogelson,[13] M. Am. Soc. C. E. The suspended

[13] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 391.

material was described as bar sand, with an effective size of 0.26 mm. The average of the runs used gave the percentage of sand as 15.97, and it was assumed that for 15% of sand the loss of head due to the presence of the sand would be proportional to the percentage of solids present, or $\frac{15}{15.97}$ times the observed loss.

Curves (3), 15%, 20%, and 25% sand, were computed from the data presented in Table 1 of the paper, the curves being then extended to indicate the rise in loss of head as the blocking-off points were approached. The blocking-off points were obtained from a smooth curve plotted as already explained, Curve (4), 15% sand, was plotted from data published by Mr. Vogelson[14] adjusted in the manner described for Curve (2), 15% sand. The analysis of the sand was as given by Mr. Vogelson.[15] The data for Curves (5), 15%, 20%, and 25% gravel, were taken from Table 1 of the paper and reduced by the foregoing method.

These are the only tests on 4-in. pipe that have come to the writer's attention, in which sufficient data have been published to make it possible to include them in the diagram, which is now sufficiently complete to warrant certain conclusions. Other data with reference to other pipe diameters might be cited to confirm the conclusions qualitatively although they have not been in sufficient detail to warrant the construction of special diagrams for those sizes.

The following conclusions may be drawn as a result of the study of Fig. 10:

(*a*) For each variety and each percentage of sand, there is a blocking-off velocity below which the pipe clogs and the flow is shut off. The heavier the percentage of sand, or the coarser the material, the higher will be the blocking-off velocity.

(*b*) With a fixed percentage of a given material the loss of head decreases as the velocity increases from the blocking-off velocity to some point where the total friction is a minimum. The velocity corresponding to the minimum value of the friction loss increases with increase in the percentage of material and with the coarseness of the material.

(*c*) For velocities higher than those corresponding to the lowest friction losses, the loss-of-head curves approach the curve for water only for a certain distance, and then become substantially parallel to that curve. (Curve (1), 15% loess, and other data not published herein, indicate that with very high velocities the loss-of-head curves diverge from the water curve).

(*d*) A study of the curves and the effective sizes of the materials indicates that the effective size may not be a suitable measure of the coarseness of the material transported. This appears reasonable as the effective size is determined by the finest 10% of the material. The curves show that the effect of 15% gravel is much greater than that of 25% sand; or, in other words, that a small proportion of coarse material may have a greater effect that a large proportion of fine material. It may develop that a better index will be the size of opening in a sieve on which 25% of the coarsest material would be retained.

[14] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 396.
[15] *Loc. cit.*, p. 395.

In practical dredging operations the muds and silts are well distributed throughout the entire cross-section of the pipe line whereas the sands and gravels concentrate near the bottom. Attention is particularly invited to the author's lucid description of the manner in which the material was moved through the transparent sections of the pipe during the experiments reported. These observations made on the 4-in. model are believed to apply qualitatively to the full-sized dredge pipes. For best results in practical dredging operations, the type of flow should be as indicated in Curves (3), 15%, 20%, and 25% sand, with velocities of 8.0 ft per sec, or more. For loess the velocities may be 6.0 ft, or less; and for gravel they should exceed 8.5 ft by an undetermined margin.

In an effort to find a practical working formula to be used in dredge design, the writer, in 1915,[16] suggested the following:

$$h = h_w + h_s = h_w + A\,N \qquad (10)$$

in which (in the notation of the paper): h = the total loss of head due to the flow of a mixture of sand and water per 1 000 ft of pipe line; h_w = the loss due to the flow of water only at the same velocity; h_s = the excess loss due to the presence of the sand; A = a factor depending upon the character of the sand and the diameter of the pipe line; and, N = the percentage of sand present in the mixture.

This formula puts into algebraic form a rule suggested by Miss Blatch[17] for determining the loss of head at the "economical velocity" and the rule given by Mr. Hazen[18] for transporting sand in 3-in. to 6-in. pipes. For conditions of flow corresponding to the best dredging range (velocities of 8 ft per sec to 12 ft

TABLE 4.—Values of Terms in Equation (10)

Mean velocity, V, in feet per second	Percentage of sand, N	(a) For Conditions of Flow Corresponding to Best Dredging Range				(b) Comparison of Computed and Observed Values				
		Total loss, h	Loss, h_w, due to water flow	Loss, h_s, due to sand	Factor A	Loss, h_w, due to water flow	Loss, h_s (=2.14 N), due to sand	Total Loss, h: Computed	Total Loss, h: Observed	Percentage error
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
8	25	109	54	55	2.20	54.0	53.5	107.5	109.0	−1
	15	90	54	36	2.40	54.0	32.1	86.1	90.0	−4
10	25	133	82	51	2.04	82.0	53.5	135.5	133.0	+2
	15	114	82	32	2.13	82.0	32.1	114.1	114.0	0
12	25	163	115	48	1.92	115.0	53.5	168.5	163.0	+3
	15	147	115	32	2.13	115.0	32.1	147.1	147.0	0

per sec, in Fig. 10), the values in Table 4(*a*), for the terms in Equation (10) may be taken from Fig. 10, or computed. The mean value of A (Column (5) Table 4(*a*)) is 2.14. If this value is assumed as correct and if the losses of head are computed by Equation (10), the comparison between the computed and observed values will be as given in Table 4(*b*).

[16] *Professional Memoirs*, Corps of Engrs., U. S. Army, March–April, 1915.
[17] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 400.
[18] American Civil Engineers' Pocket Book, 1912 Edition, p. 935.

The relatively small errors that would result, in this model test, from the assumption that, within the range simulating practical dredging velocities, the value of A is dependent solely upon the character of the material and the diameter of the pipe line, are most encouraging. It is hoped that additional data for testing this method will become available, and that, if the method is found to have merit, the working values for A will be established.

Practical dredging velocities have increased greatly in recent years due to the development of better machinery and other equipment. Fig. 10 shows how higher velocities make it possible to transport greater percentages of sand. In a 24-in. pipe line a velocity of 24 ft should be within the range of modern equipment and should be at the call of the crew whenever the need for it arises. Lower velocities would be adequate for pipe lines of smaller diameters. In a 1-in. brass pipe a velocity of 5 ft per sec was shown by Miss Blatch to be ample for sand having a median diameter of about 0.63 mm. In a 3-in. pipe with sand having a median diameter of about 0.65 mm, Mr. Vogelson found that a velocity of 8.5 was sufficient; and, with the same sand in a 4-in. pipe a velocity of 7.0 ft per sec was inadequate. Precise observational data for full-sized dredge pipes are not available and, as stated by the author, they are probably not obtainable.

With respect to the diameter of the pipe lines it is known that, for the small sizes, the values of A vary rapidly with the diameter. It is doubtful, however, whether the variation on this account for dredge pipes of 12 to 34 in. in diameter are large. A comparison of Miss Blatch's experiment with a 1-in. brass pipe and Mr. Vogelson's experiment with a 3-in. pipe gives the following:

Description	One-inch brass pipe	Three-inch iron pipe
Suitable velocity, V, in feet per second	5.0	8.5
Head Losses in Feet:		
h	280	141
h_w	100	87
h_s	180	54
Percentage of sand	17.78	17.78
Factor A	10.10	3.04

The best available information is to the effect that, with similar sand, the values of A for pipe lines 12 in. to 34 in. in diameter would be between 1.0 and 1.5.

With respect to the effect of coarseness of the sand upon the values of A, the writer knows of no better information than that shown in Fig. 10, for 4-in. pipes. For full-sized dredge pipes the values are relatively small and the range of values is correspondingly small, provided always that the velocities are greater than those for the minimum friction losses on the head-velocity curves.

As long as a dredge can pick up a full load for its pipe line the most economical velocity will be the maximum obtainable with the equipment. An increase in velocity results in the transportation of a larger volume of mixture and the percentage of solids in the mixture may also be increased. The increase in power required to produce the extra velocity will cost much less than the value of the increase in pay work.

This paper is a valuable and timely contribution to the subject. If it encourages the publication of additional data and promotes a better understanding of the problem, future experimental work can be aimed at the solution of the remaining essential problems and progress in the direction of the final solution will be greatly facilitated.

DAVID L. NEUMAN,[19] M. AM. SOC. C. E. (by letter).[19a]—The definition of "economical velocity" quoted, but not used, by Mr. Howard is of little value in dredge operation. The definition given is that (see "The Economical Velocity for Sand Transportation") "velocity at which any given volume of sand per hour can be transported through a given length of pipe with the least expenditure of power per unit volume of sand transported * * * ." Such "economical velocity" will not result in minimum unit total costs. Fuel costs are a minor part of the total costs of dredging operations. For forty-five pipe-line, cutter-type, hydraulic dredges, varying in size from 12 in. to 30 in., the average fuel costs for 1937 was 13.3%; the average for 12-in. dredges was 8.3%; 20-in. dredges, 11.2%; and, 28-in. dredges, 17 per cent. Generally, minimum unit costs are obtained with maximum output per pumping hour.

For many materials there is a rather narrow range of velocities in which maximum output is obtained. In a few instances it has been observed that a change of average velocity of 1.5 ft per sec, above or below the velocity producing the maximum output, would reduce the output as much as 40 per cent. This change in output is due to the change of velocity in the suction pipe. In soft or light material, especially, a small change in suction velocity may strongly effect the output. Theoretical considerations indicate that such results could be expected. The "economical velocity," as previously defined, would fall below the range producing maximum output, and the fuel savings through operation at the "economical velocity" would not offset the reduction in output caused by it. However, within the range of velocities producing maximum outputs fuel economy may be important. Many dredge-masters operate under the theory that maximum output is obtained with maximum obtainable discharge velocities. Frequently, outputs with existing dredges can be increased materially by decreasing the discharge velocities.

The discharge velocities must be sufficiently high to carry off, without danger of blocking the line, all the material that can be picked up on the suction side of the pump. Such velocities should be of Mr. Howard's third type of flow in which material is moving over the entire cross-sectional area of the discharge line. Average discharge-line loadings vary from about 12% to 20%; for short periods of seconds duration, loadings as great as 50% solids occur. The values are apparent percentages, which include the water in the voids for the material in place on the bottom. Indications are that for average loading in sand the third type of flow is obtained for values of velocity greater than 15 ft per sec in a 12-in. line, and greater than 19 ft per sec for a 24-in. line. In a 20-in. discharge line, it has been noted that even at a velocity of 22 ft per sec the line could be loaded so heavily with sand that some of the material could be heard moving

[19] Maj., Corps of Engrs., U. S. Army, Grosse Pointe Village, Mich.

[19a] Received by the Secretary October 28, 1938.

along the bottom of the pipe. Table 1(*a*), for sand in a 4-in. line, indicates that at 12 ft per sec the loss of head, in feet of mixture, between 10% and 30% solids, was not increased by an increase of percentage of solids. Pipe-line blocking, even in coarse material, including boulders with mean diameters up to about 70% of the pipe diameter, can be avoided for the most part by the use of sufficiently higher discharge velocities. Velocities of 21 ft per sec for a 12-in. line to 30 ft per sec for a 30-in. line are suggested. In dredge design for increased higher discharge velocities, the suction diameter must be correspondingly increased to avoid the reduction of the picking up power of the suction line through too high suction velocity. Practical experience first indicated the need of small suction velocities with relatively high discharge velocities, and dredge design in recent years has followed the trend of increasingly larger suction diameters compared to discharge diameter with increased pump power for the higher discharge velocities. Apparently, minimum suction velocities have been taken at from 12 ft per sec to 14 ft per sec. It is believed that suction velocities might be safely reduced to 8 ft per sec and the output increased by such reduction.

The results of Mr. Howard's investigation could be used effectively if comparative results were obtainable for large size pipe such as that used for dredge discharge lines. Dependable and accurate gages permitting a continuous record of discharge velocities and percentage of solids passing through the discharge line have been developed and used since 1937 in connection with dredging operations. With the use of such gages in dredging operations results similar to those published by Mr. Howard are obtainable. With such gages installed on a dredge the range of discharge velocities producing maximum output and minimum unit costs can be determined on the site of the work, during actual dredging operation, without interference with such operation.

A most worth while investigation pertaining to obtaining increased output and decreased unit costs in hydraulic dredging can be secured through investigation of the action around the suction-mouth opening and in the suction pipe itself. Such investigation might well include the determination of favorable suction velocities for picking up various materials, head losses in the suction line, effective design of the suction-mouth opening, and the relation of suction-line vacuum and velocities to picking up and transporting ability. Such studies can best be performed in the laboratory where visual observation of suction-line action is possible.

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* Louis C. Hill, Past-President, Am. Soc. C. E., died November 5, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS

COMING MEETINGS

Board of Direction Meetings

January 16–17, 1939:

A Quarterly Meeting will be held in New York, N. Y.

ANNUAL MEETING

NEW YORK, N. Y.

January 18, 19, 20, 1939

The Reading Room of the Society is open from 9:00 A.M. to 5:00 P.M. every day, except Saturdays when it is closed at 12:00 A.M. It is closed all day on Sundays and holidays.

Members, particularly those from out of town, are cordially invited to use this room on their visits to New York, to have their mail addressed there, and to utilize it as a place for meeting others. There is an ample file of current periodicals, the latest technical books, and the room is well supplied with writing tables.